



185.A83 Machine Learning for Health Informatics 2016S, VU, 2.0 h, 3.0 ECTS
Week 20 18.05.2016 17:00-20:00

Probabilistic Graphical Models Part 2: From Bayesian Networks to Graph Bandits

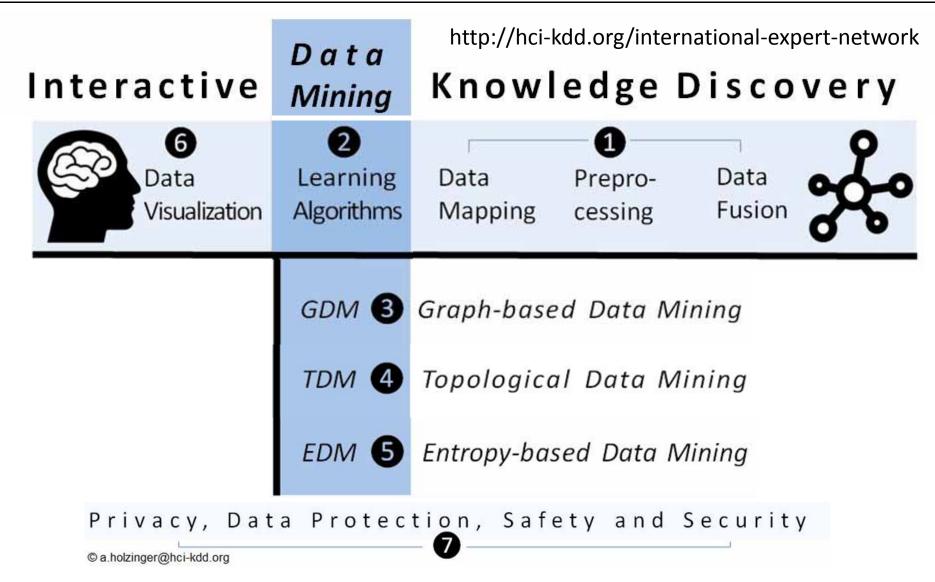
a.holzinger@hci-kdd.org

http://hci-kdd.org/machine-learning-for-health-informatics-course









Holzinger, A. 2014. Trends in Interactive Knowledge Discovery for Personalized Medicine: Cognitive Science meets Machine Learning. IEEE Intelligent Informatics Bulletin, 15, (1), 6-14.

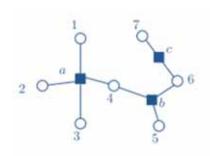


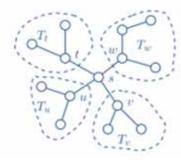
- 1) Graphical Models and Decision Making
- 2) Bayesian Networks
- 3) Machine Learning on Graphs
- 4) Little Excursus: What is similarity?
- 5) Probabilistic Topic Models
- 6) Graph Bandits (a very hot topic!)

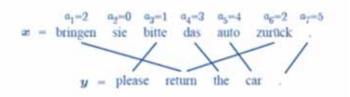


Let us start with a warm-up Quiz (solutions -> last page)









1

-

3

$$E(U \mid d) = \sum_{x_1, \dots, x_n} P(x_1, \dots, x_n \mid d) U(x_1, \dots, x_n, d)$$

4

h, = The identity of ORGANISM-1 is streptococcus

h₂ = PATIENT-1 is febrile

h₃ = The name of PATIENT-1 is John Jones

CF[h₁,E] = .8 : There is strongly suggestive evidence (.8) that

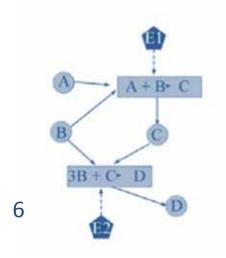
the identity of ORGANISM-1 is streptococcus

 $CF[h_2, E] = -.3$: There is weakly suggestive evidence (.3) that

PATIENT-1 is not febrile

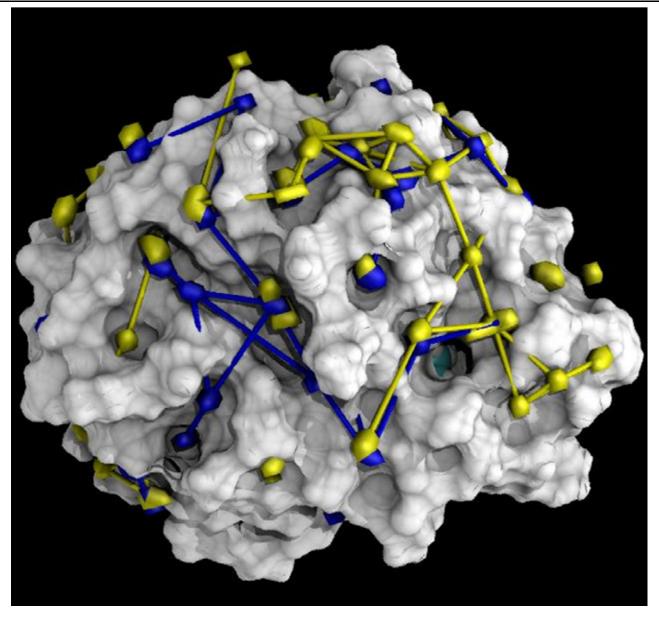
CF[h₃,E] = +1 : It is definite (1) that the name of PATIENT-1 is

John Jones



5

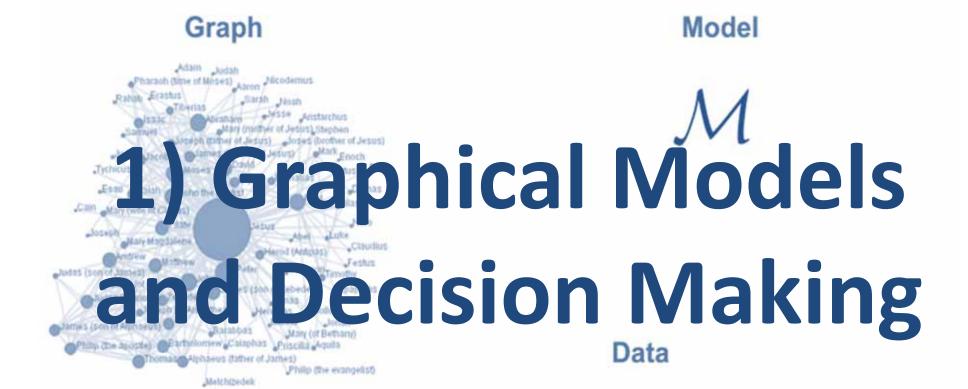




http://sbcb.bioch.ox.ac.uk/users/oliver/software/



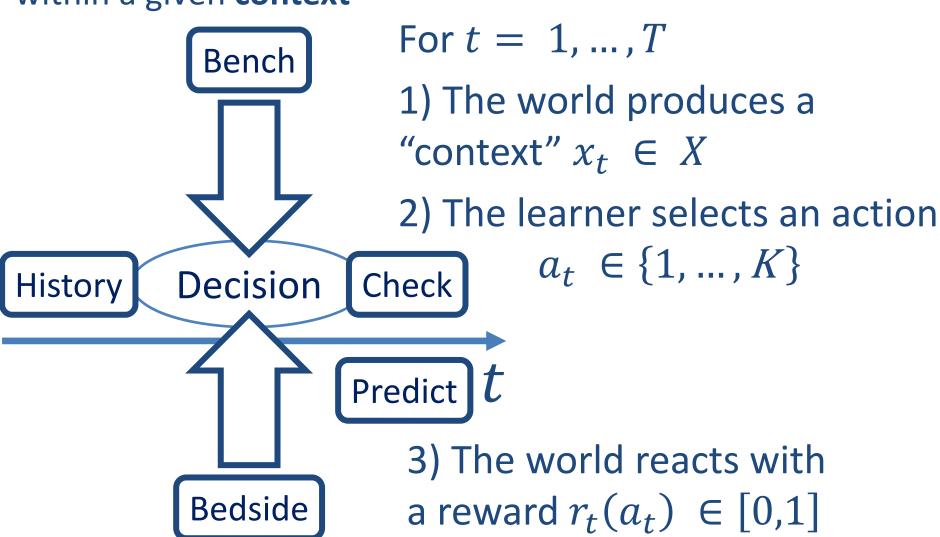




$$\mathcal{D} \equiv \{X_1^{(i)}, X_2^{(i)}, ..., X_m^{(i)}\}_{i=1}^N$$



Goal: Learn an optimal policy for selecting best actions within a given context





- Key Idea: Conditional independence assumptions are very useful – however: Naïve Bayes is extreme!
- X is conditionally independent of Y, given Z, if the P(X) governing X is independent of value Y, given value of Z:

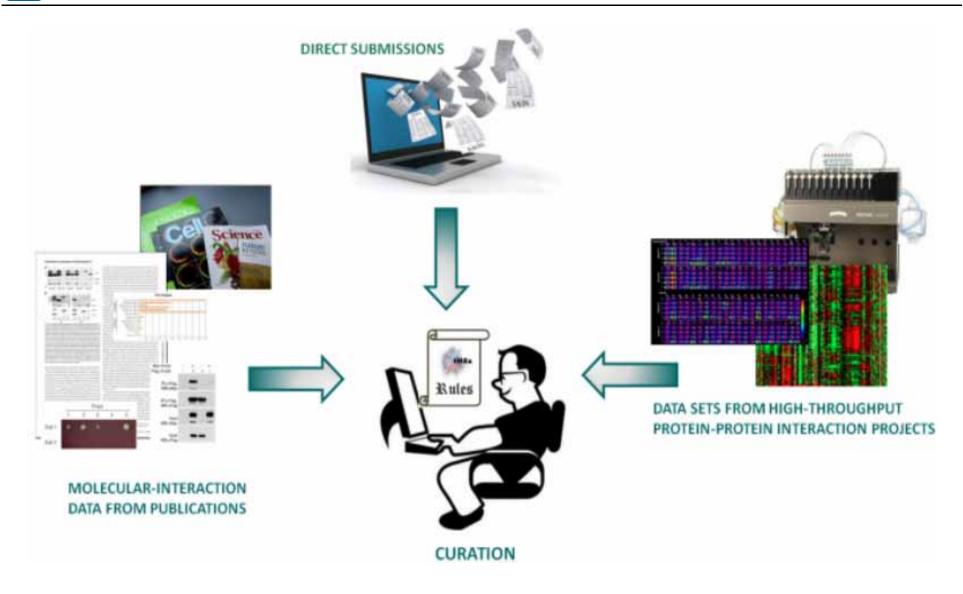
$$(\forall i,j,k)P(X=x_i|Y=y_j,Z=z_k)=P(X=x_i|Z=z_k)$$
 can be abbr. with $P(X|Y,Z)=P(X|Z)$

- Graphical models express sets of conditional independence assumptions via graph structure
- The graph structure plus associated parameters define joint probability distribution over the set of variables



Where do the data come from?



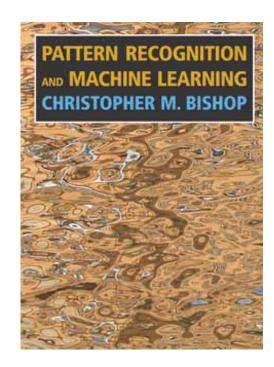


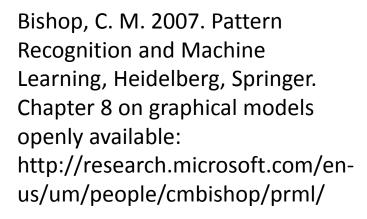
http://www.ebi.ac.uk/intact/

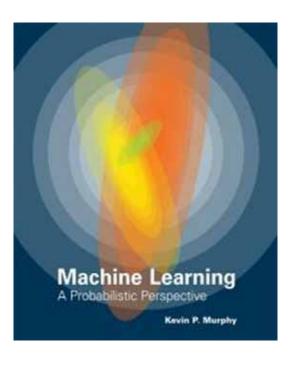


- Medicine is an extremely complex application domain – dealing most of the time with uncertainties -> probable information!
- When we have big data but little knowledge automatic ML can help to gain insight:
- Structure learning and prediction in large-scale biomedical networks with probabilistic graphical models
- If we have little data and deal with NP-hard problems we still need the human-in-the-loop



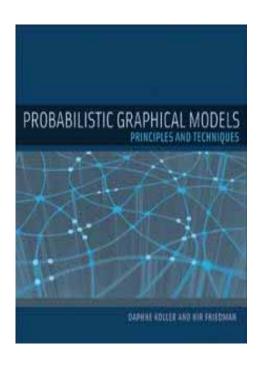






Murphy, K. P. 2012.

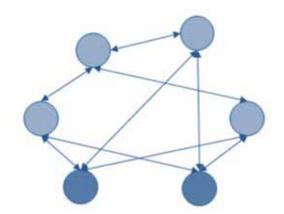
Machine learning: a probabilistic perspective, MIT press. Chapter 26 (pp. 907) – Graphical model structure learning



Koller, D. & Friedman, N. 2009. Probabilistic graphical models: principles and techniques, MIT press.

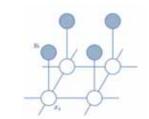
Three types of Probabilistic Graphical Models

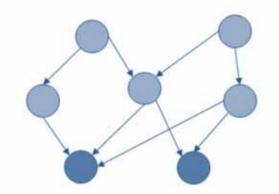




Undirected: Markov random fields, useful e.g. for computer vision (Details: Murphy 19)

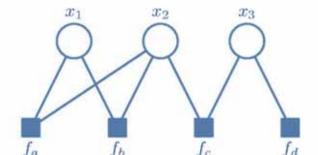
$$P(X) = \frac{1}{Z} \exp\left(\sum_{ij} W_{ij} x_i x_j + \sum_i x_i b_i\right)$$





Directed: Bayes Nets, useful for designing models (Details: Murphy 10)

$$p(\mathbf{x}) = \prod_{k=1}^{K} p(x_k | \mathbf{pa}_k)$$



Factored: useful for inference/learning

$$p(\mathbf{x}) = \prod_{s} f_s(\mathbf{x}_s)$$





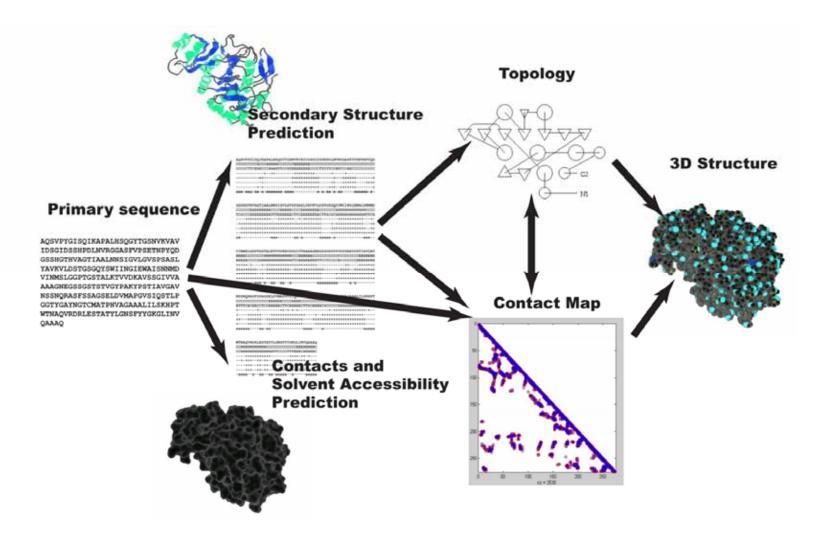
What is the advantage of factor graphs?

| | Dependency | Efficient Inference | Usage |
|-------------------|------------|------------------------|--|
| Bayesian Networks | Yes | Somewhat | Ancestral Generative Process |
| Markov Networks | Yes | No | Local Couplings and Potentials |
| Factor Graphs | No | Yes | Efficient, distributed inference |

Table credit to Ralf Herbrich, Amazon







Baldi, P. & Pollastri, G. 2003. The principled design of large-scale recursive neural network architectures--dag-rnns and the protein structure prediction problem. The Journal of Machine Learning Research, 4, 575-602.





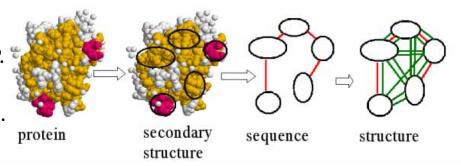
- Hypothesis: most biological functions involve the interactions between many proteins, and the complexity of living systems arises as a result of such interactions.
- In this context, the problem of inferring a global protein network for a given organism,
- using all (genomic) data of the organism,
- is one of the main challenges in computational biology

Yamanishi, Y., Vert, J.-P. & Kanehisa, M. 2004. Protein network inference from multiple genomic data: a supervised approach. Bioinformatics, 20, (suppl 1), i363-i370.

Problem: Is Graph Isomorphism NP-complete?



Borgwardt, K. M., Ong, C. S., Schönauer, S., Vishwanathan, S., Smola, A. J. & Kriegel, H.-P. 2005. Protein function prediction via graph kernels. Bioinformatics, 21, (suppl 1), i47-i56.



- Important for health informatics: Discovering relationships between biological components
- Unsolved problem in computer science:
- Can the graph isomorphism problem be solved in polynomial time?
 - So far, no polynomial time algorithm is known.
 - It is also not known if it is NP-complete
 - We know that subgraph-isomorphism is NP-complete

Example: Protein Network Inference



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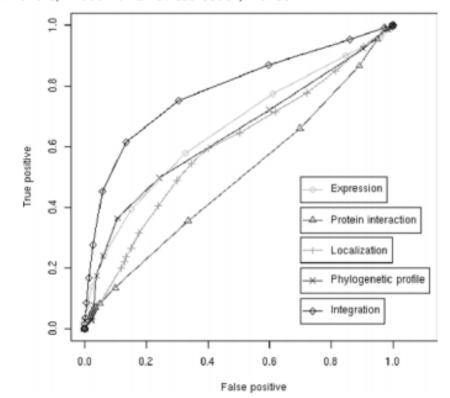
Vol. 20 Suppl. 1 2004, pages i363-i370 DOI: 10.1093/bioinformatics/bth910



Protein network inference from multiple genomic data: a supervised approach

Y. Yamanishi^{1,*}, J.-P. Vert² and M. Kanehisa¹

¹ Bioinformatics Center, Institute for Chemical Research, Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan and ² Computational Biology group, Ecole des Mines de Paris, 35 rue Saint-Honoré, 77305 Fontainebleau cedex, France



 $K_{\rm exp}$ (Expression)

 K_{ppi} (Protein interaction)

 K_{loc} (Localization)

K_{phy} (Phylogenetic profile)

 $K_{\text{exp}} + K_{\text{ppi}} + K_{\text{loc}} + K_{\text{phy}}$ (Integration)

Example: Data fusion and Protein Annotation



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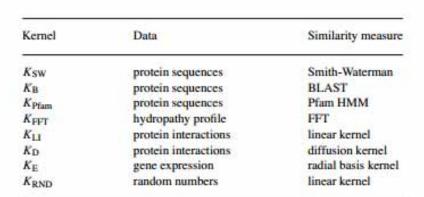
Val. 20 no. 16 2004, pages 2626–2635 doi:10.1093/bioinformatics/bth294

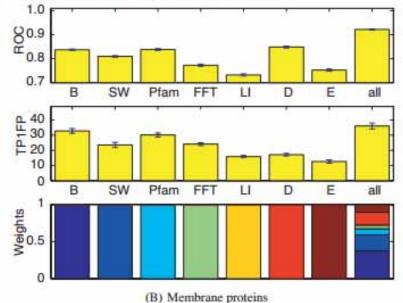


A statistical framework for genomic data fusion

Gert R. G. Lanckriet¹, Tijl De Bie³, Nello Cristianini⁴, Michael I. Jordan² and William Stafford Noble^{5,*}

¹Department of Electrical Engineering and Computer Science, ²Division of Computer Science, Department of Statistics, University of California, Berkeley 94720, USA, ³Department of Electrical Engineering, ESAT-SCD, Katholieke Universiteit Leuven 3001, Belgium, ⁴Department of Statistics, University of California, Davis 95618, USA and ⁵Department of Genome Sciences, University of Washington, Seattle 98195, USA





Lanckriet, G. R., De Bie, T., Cristianini, N., Jordan, M. I. & Noble, W. S. 2004. A statistical framework for genomic data fusion. Bioinformatics, 20, (16), 2626-2635.





2) BayesianNetworks"Bayes' Nets"





- is a probabilistic model, consisting of two parts:
- 1) a dependency structure and
- 2) local probability models.

$$p(x_1,...,x_n) = \prod_{i=1}^n p(x_i | Pa(x_i))$$

Where $Pa(x_i)$ are the parents of x_i

BN inherently model the <u>uncertainty in the data.</u> They are a successful marriage between probability theory and graph theory; allow to model a multidimensional probability distribution in a sparse way by searching independency relations in the data. Furthermore this model allows different strategies to integrate two data sources.

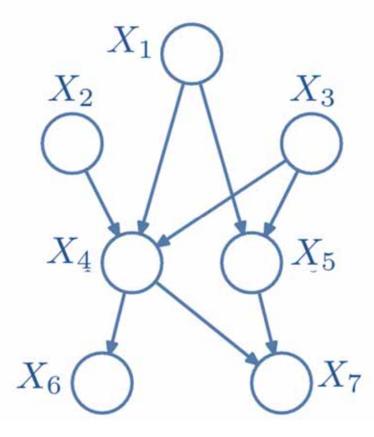
Pearl, J. (1988) Probabilistic reasoning in intelligent systems: networks of plausible inference. San Francisco, Morgan Kaufmann.



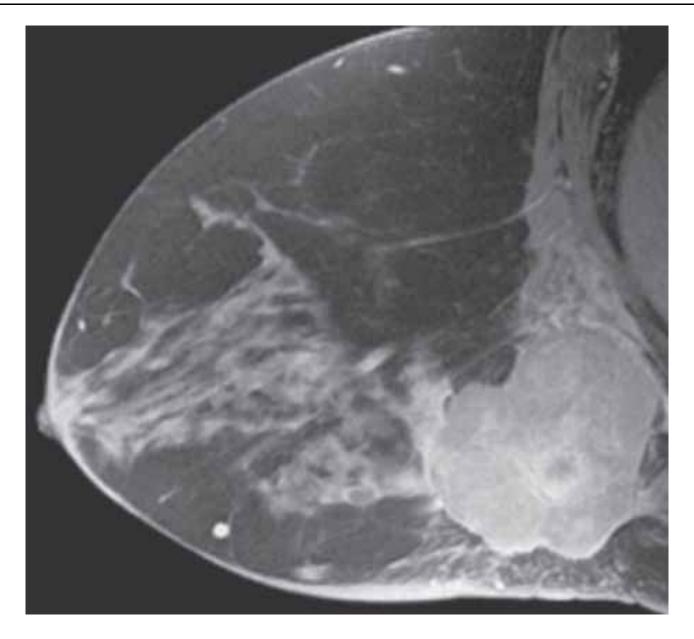
$$p(X_1, \dots, X_7) =$$

$$p(X_1)p(X_2)p(X_3)p(X_4|X_1, X_2, X_3) \cdot$$

$$p(X_5|X_1, X_3)p(X_6|X_4)p(X_7|X_4, X_5)$$





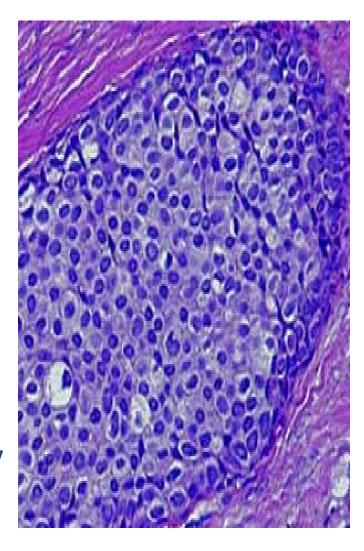


Overmoyer, B. A., Lee, J. M. & Lerwill, M. F. (2011) Case 17-2011 A 49-Year-Old Woman with a Mass in the Breast and Overlying Skin Changes. New England Journal of Medicine, 364, 23, 2246-2254.



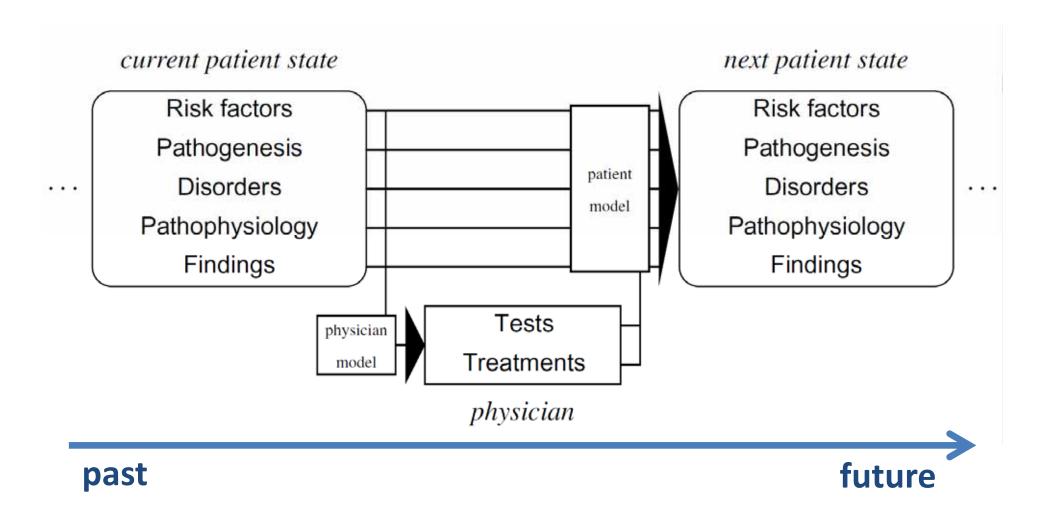


- = the prediction of the future course of a disease conditional on the patient's history and a projected treatment strategy
- Danger: probable Information!
- Therefore valid prognostic models can be of great benefit for clinical decision making and of great value to the patient, e.g., for notification and quality of-life decisions



Knaus, W. A., Wagner, D. P. & Lynn, J. (1991) Short-term mortality predictions for critically ill hospitalized adults: science and ethics. *Science*, *254*, *5030*, *389*.





van Gerven, M. A. J., Taal, B. G. & Lucas, P. J. F. (2008) Dynamic Bayesian networks as prognostic models for clinical patient management. *Journal of Biomedical Informatics*, 41, 4, 515-529.



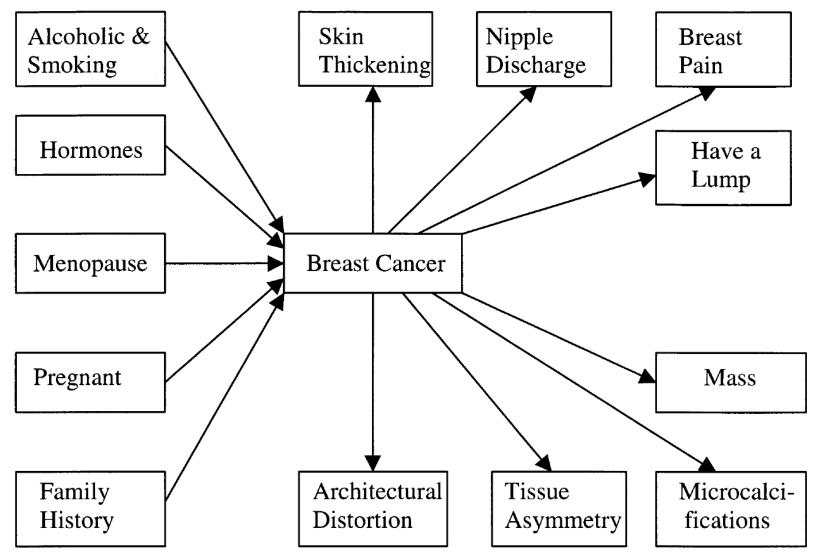
Example: Breast cancer - Probability Table



| Category | Node description | State description |
|-------------------------------|---|--|
| Diagnosis | Breast cancer | Present, absent. |
| Clinical history | Habit of drinking alcoholic beverages and smoking | Yes, no. |
| • | Taking female hormones | Yes, no. |
| | Have gone through menopause | Yes, no. |
| | Have ever been pregnant | Yes, no. |
| | Family member has breast cancer | Yes, no. |
| Physical find- ings | Nipple discharge | Yes, no. |
| | Skin thickening | Yes, no. |
| | Breast pain | Yes, no. |
| | Have a lump(s) | Yes, no. |
| Mammo- graphic findings | Architectural distortion | Present, absent. |
| J | Mass | Score from one to three, score from four to five, absent |
| | Microcalcification cluster | Score from one to three, score from four to five, absent |
| | Asymmetry | Present, absent. |

Wang, X. H., et al. (1999) Computer-assisted diagnosis of breast cancer using a data-driven Bayesian belief network. *International Journal of Medical Informatics*, 54, 2, 115-126.





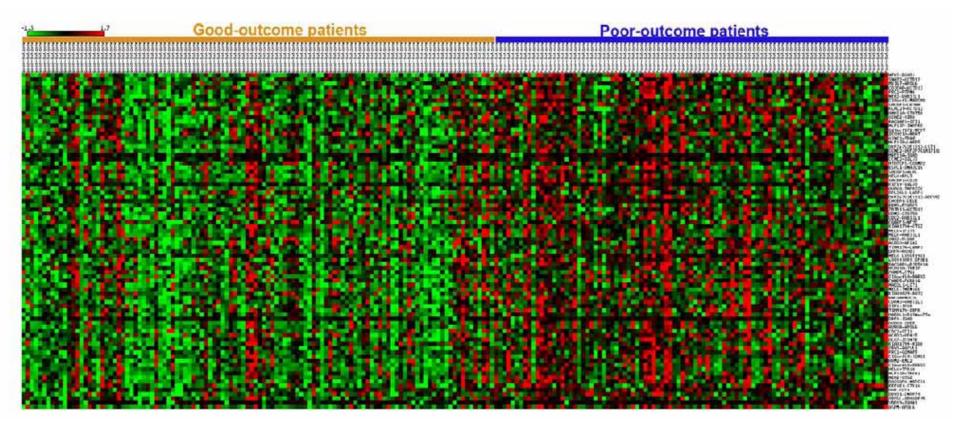
Wang, X. H., et al. (1999) Computer-assisted diagnosis of breast cancer using a data-driven Bayesian belief network. *International Journal of Medical Informatics*, *54*, *2*, *115-126*.



10 years later: Integration of microarray data



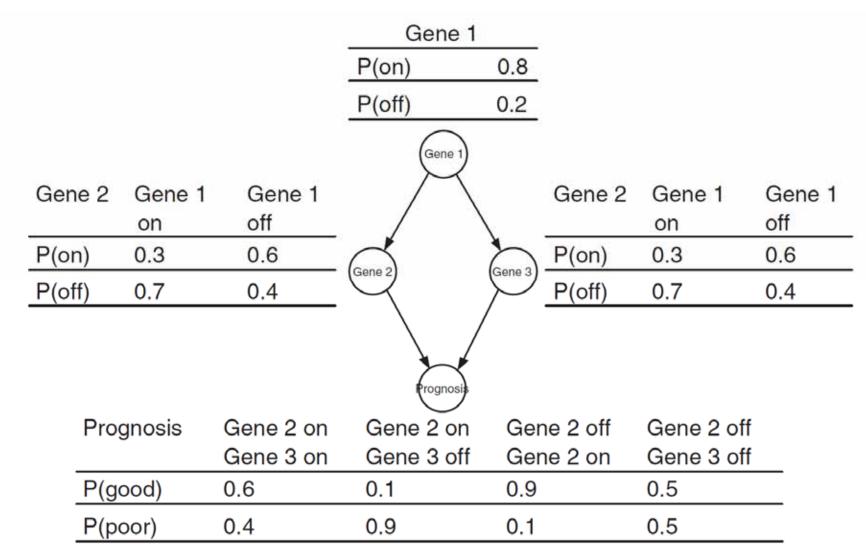
- Integrating microarray data from multiple studies to increase sample size;
- = approach to the development of more robust prognostic tests



Xu, L., Tan, A., Winslow, R. & Geman, D. (2008) Merging microarray data from separate breast cancer studies provides a robust prognostic test. *BMC Bioinformatics*, 9, 1, 125-139.

Example: BN with four binary variables

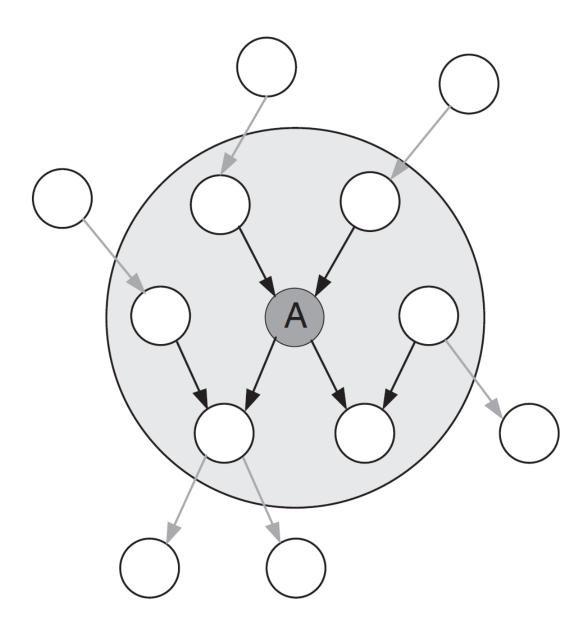




Gevaert, O., Smet, F. D., Timmerman, D., Moreau, Y. & Moor, B. D. (2006) Predicting the prognosis of breast cancer by integrating clinical and microarray data with Bayesian networks. *Bioinformatics*, 22, 14, 184-190.



Gevaert, O., Smet, F. D., Timmerman, D., Moreau, Y. & Moor, B. D. (2006) Predicting the prognosis of breast cancer by integrating clinical and microarray data with Bayesian networks. *Bioinformatics*, 22, 14, 184-190.





- First the structure is learned using a <u>search strategy</u>.
- Since the number of possible structures <u>increases</u> super exponentially with the number of variables,
- the well-known greedy search algorithm K2 can be used in combination with the <u>Bayesian Dirichlet (BD) scoring metric</u>:

$$p(S|D) \propto p(S) \prod_{i=1}^{n} \prod_{j=1}^{q_i} \left[\frac{\Gamma(N'_{ij})}{\Gamma(N'_{ij} + N_{ij})} \prod_{k=1}^{r_i} \frac{\Gamma(N'_{ijk} + N_{ijk})}{\Gamma(N'_{ijk})} \right]$$

 N_{ijk} ... number of cases in the data set D having variable i in state k associated with the j-th instantiation of its parents in current structure S.



Dependency Structure – first step (2/2)



- Next, N_{ij} is calculated by summing over all states of a variable:
- $N_{ij} = \sum_{k=1}^{r_i} N_{ijk} \cdot N'_{ijk}$ and N'_{ij} have similar meanings but refer to prior knowledge for the parameters.
- When no knowledge is available they are estimated using $N_{ijk} = N/(r_i q_i)$
- with N the equivalent sample size,
- r_i the number of states of variable i and
- q_i the number of instantiations of the parents of variable i.
- $\Gamma(.)$ corresponds to the gamma distribution.
- Finally p(S) is the prior probability of the structure.
- p(S) is calculated by:
- $p(S) = \prod_{i=1}^{n} \prod_{l=1}^{p_i} p(l_i \to x_i) \prod_{m_i=1}^{o_i} p(m_i x_i)$
- with p_i the number of parents of variable x_i and o_i all the variables that are not a parent of x_i .
- Next, $p(a \rightarrow b)$ is the probability that there is an edge from a to b while p(ab) is the inverse, i.e. the probability that there is no edge from a to b

Parameter learning -> second step



- Estimating the parameters of the local probability models corresponding with the dependency structure.
- CPTs are used to model these local probability models.
- For each variable and instantiation of its parents there exists a CPT that consists of a set of parameters.
- Each set of parameters was given a uniform Dirichlet prior:

$$p(\theta_{ij}|S) = Dir(\theta_{ij}|N'_{ij1}, \dots, N'_{ijk}, \dots, N'_{ijr_i})$$

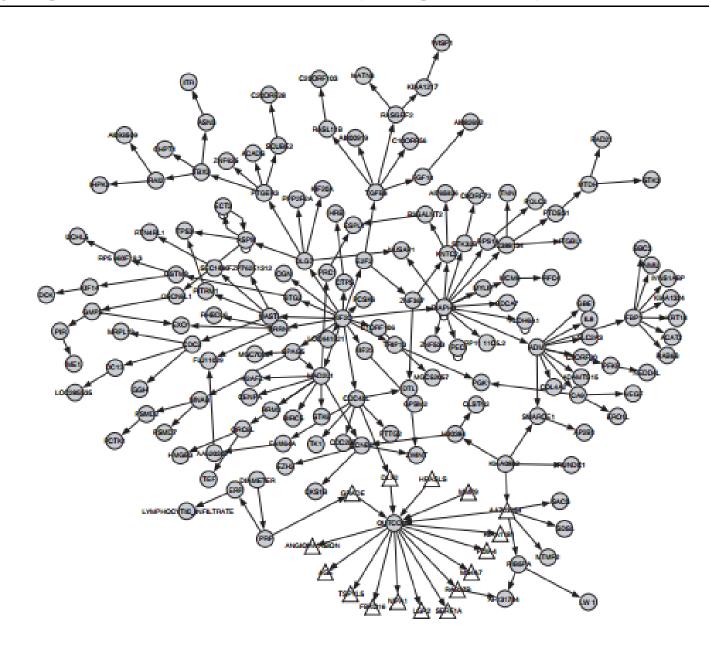
Note: With θ_{ij} a parameter set where i refers to the variable and j to the j-th instantiation of the parents in the current structure. θ_{ij} contains a probability for every value of the variable x_i given the current instantiation of the parents. Dir corresponds to the Dirichlet distribution with $(N'_{ij1}, \ldots, N'_{ijr_i})$ as parameters of this Dirichlet distribution. Parameter learning then consists of updating these Dirichlet priors with data. This is straightforward because the multinomial distribution that is used to model the data, and the Dirichlet distribution that models the prior, are conjugate distributions. This results in a Dirichlet posterior over the parameter set:

$$p(\theta_{ij}|D,S) = Dir(\theta_{ij}|N'_{ij1} + N_{ij1}, ..., N'_{ijk} + N_{ijk}, ..., N'_{ijr_i} + N_{ijr_i})$$

with N_{ijk} defined as before.

Predicting the prognosis of breast cancer (integrated a.)





Gevaert, O., Smet, F. D., Timmerman, D., Moreau, Y. & Moor, B. D. (2006) Predicting the prognosis of breast cancer by integrating clinical and microarray data with Bayesian networks. *Bioinformatics*, 22, 14, 184-190.





- For certain cases it is tractable if:
 - Just one variable is unobserved
 - We have singly connected graphs (no undirected loops -> belief propagation)
 - Assigning probability to fully observed set of variables
- Possibility: Monte Carlo Methods (generate many samples according to the Bayes Net distribution and then count the results)
- Otherwise: approximate solutions, NOTE:

Sometimes it is better to have an approximate solution to a complex problem – than a perfect solution to a simplified problem





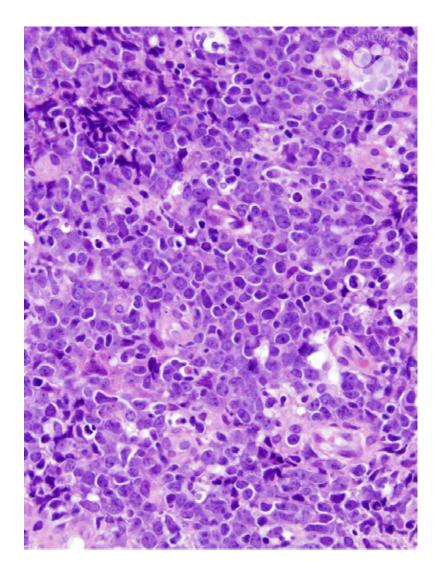
3) Machine Learning on Graphs



Example: Lymphoma is the most common blood cancer



The two main forms of lymphoma are Hodgkin lymphoma and non-Hodgkin lymphoma (NHL). Lymphoma occurs when cells of the immune system called lymphocytes, a type of white blood cell, grow and multiply uncontrollably. Cancerous lymphocytes can travel to many parts of the body, including the lymph nodes, spleen, bone marrow, blood, or other organs, and form a mass called a tumor. The body has two main types of lymphocytes that can develop into lymphomas: Blymphocytes (B-cells) and Tlymphocytes (T-cells).



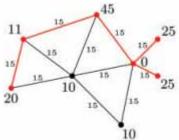
www.lymphoma.org

http://imagebank.hematology.org/



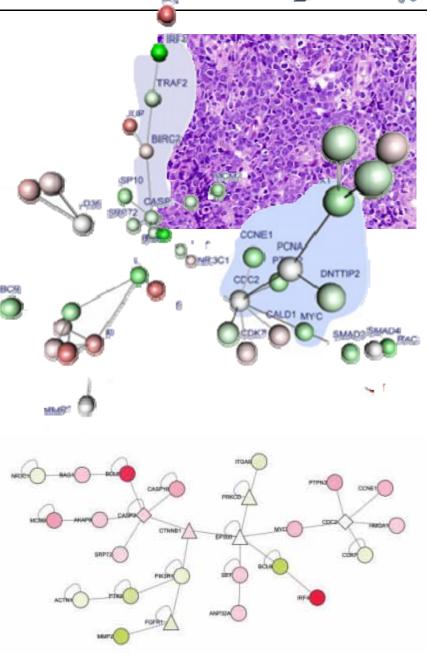


- Discover unexplored interactions in PPInetworks and gene regulatory networks
- Learn the structure
- Reconstruct the structure



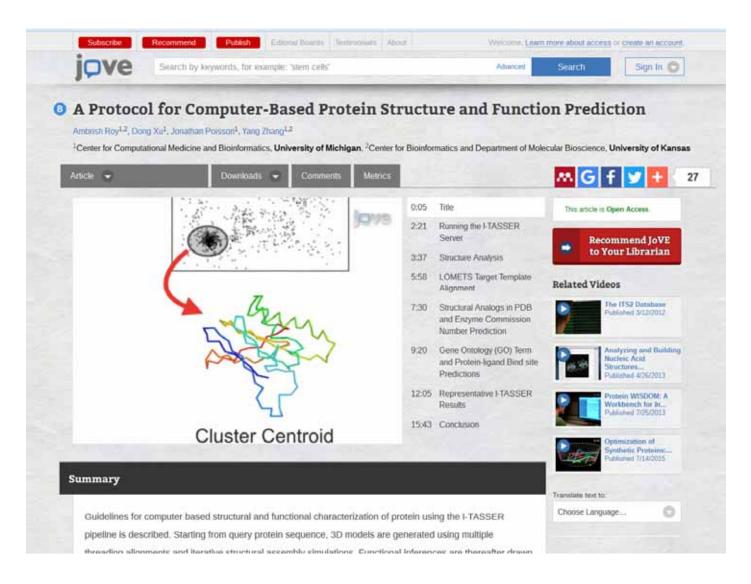
Dittrich, M. T., Klau, G. W., Rosenwald, A., Dandekar, T. & Müller, T. 2008. Identifying functional modules in protein–protein interaction networks: an integrated exact approach. Bioinformatics, 24, (13), i223-i231.

Holzinger Group









http://www.jove.com/video/3259/a-protocol-for-computer-based-protein-structure-function



Nodes: proteins

Links: physical interactions (binding)

Puzzling pattern:

Hubs tend to link to small degree nodes.

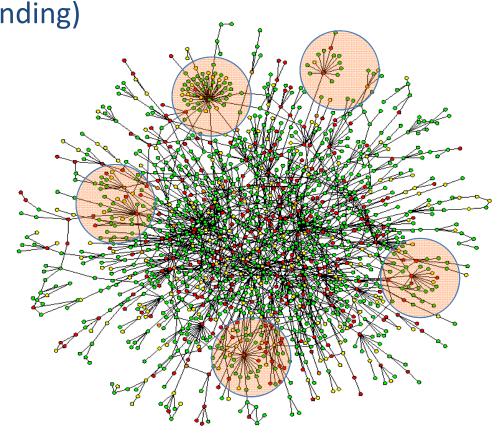
Why is this puzzling?

In a random network, the probability that a node with degree k links to a node with degree k' is:

$$p_{kk'} = \frac{kk'}{2L}$$

k≅50, k'=13, N=1,458, L=1746

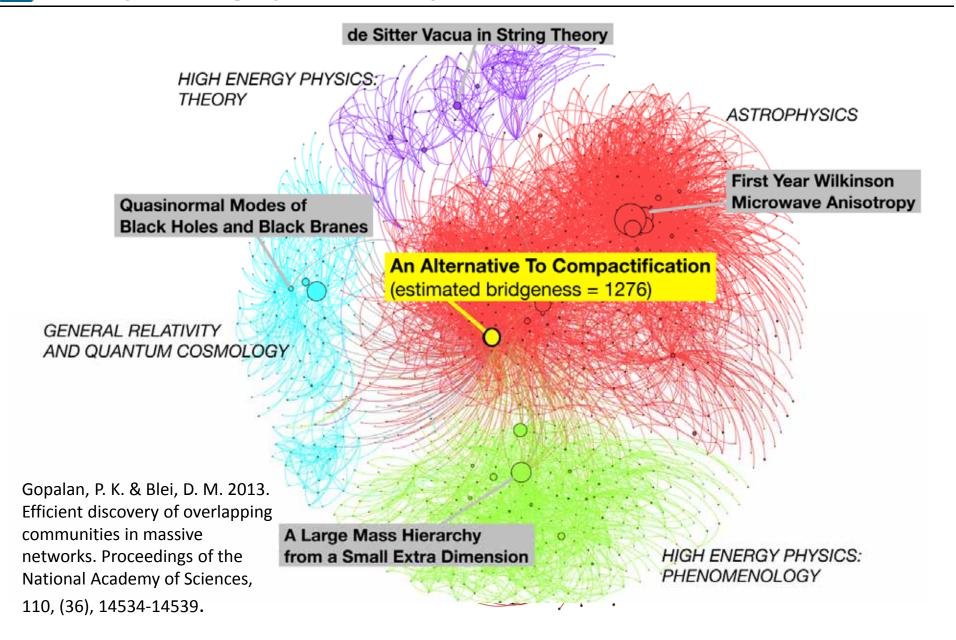
$$p_{50,13} = 0.15$$
 $p_{2,1} = 0.0004$



Jeong, H., Mason, S. P., Barabasi, A. L. & Oltvai, Z. N. 2001. Lethality and centrality in protein networks. Nature, 411, (6833), 41-42.

Example: Subgraph Discovery

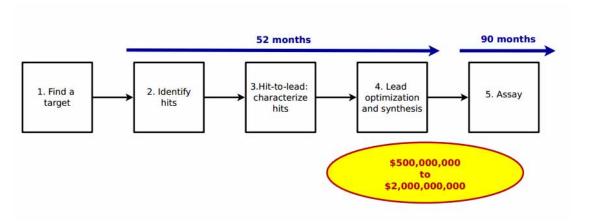








- A) Discovery of unexplored interactions
- B) Learning and Predicting the structure
- C) Reconstructing the structure
- Which joint probability distributions does a graphical model represent?
- How can we learn the parameters and structure of a graphical model?



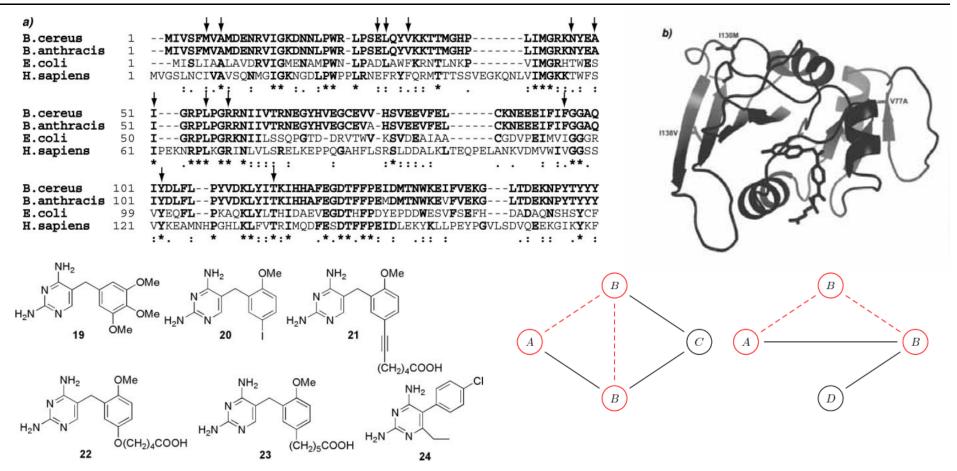
The chemical space

- 10⁶⁰ possible small organic molecules
- 10²² stars in the observable universe



Example Question: Predicting Function from Structure





How similar are two graphs? How similar is their structure? How similar are their node and edge labels?

Joska, T. M. & Anderson, A. C. 2006. Structure-activity relationships of Bacillus cereus and Bacillus anthracis dihydrofolate reductase: toward the identification of new potent drug leads. Antimicrobial agents and chemotherapy, 50, 3435-3443.



- Similar Property Principle: Molecules having similar structures should have similar activities.
- Structure-based representations: Compare molecules by comparing substructures, e.g.
 - Sets as vectors: Measure similarity by the cosine distance
 - Sets as sets: Measure similarity by the Jaccard distance
 - Sets as points: Measure similarity by Euclidean distance
- Problems: Dimensionality, Non-Euclidean cases





4) Little Excursus: What is similarity?



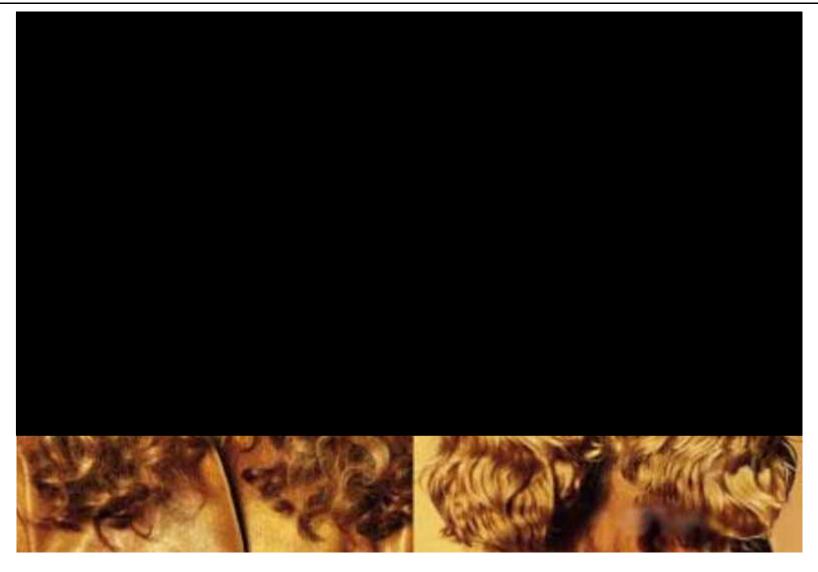
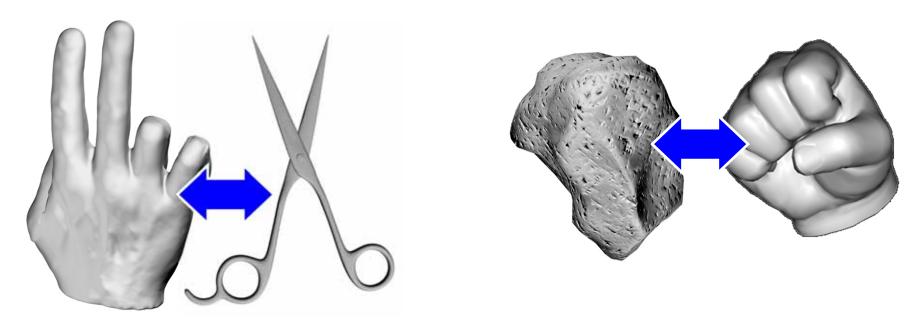


Image credit to Eamonn Keogh (2008)



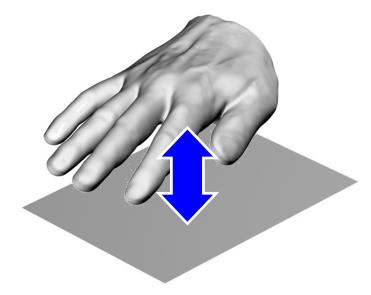




Bronstein, A. M., Bronstein, M. M. & Kimmel, R. 2008.

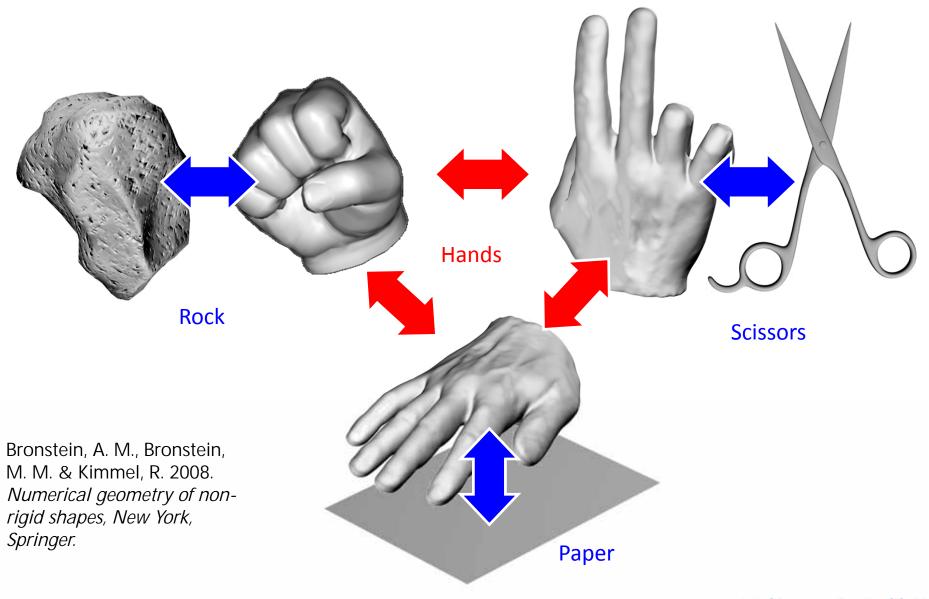
Numerical geometry of non-rigid shapes, New York,

Springer.









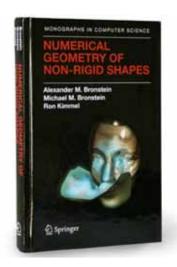


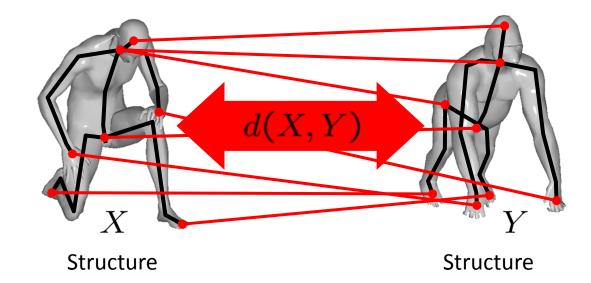
Similarity and Correspondence



Bronstein, A. M., Bronstein, M. M. & Kimmel, R. 2008. *Numerical geometry of non-rigid shapes, New York, Springer.*

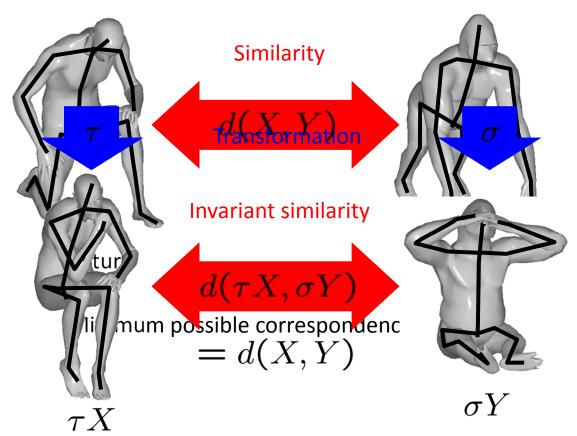
http://www.inf.usi.ch/bronstein/





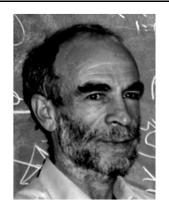
Correspondence quality = structure similarity (distortion)

Minimum possible correspondence distortion



WGromov-Hausdorff dist: finding the opt. correspondence

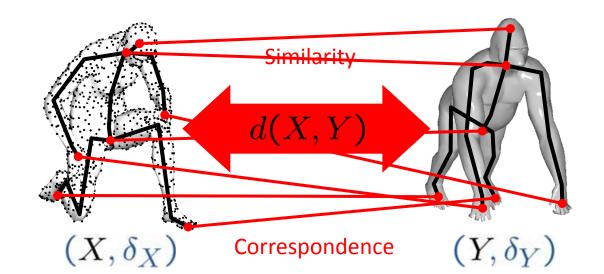




Michail Gromov (1943-)

Gromov, M. (1984) Infinite groups as geometric objects.

Metric space





Felix Hausdorff (1868-1942)

Metric space

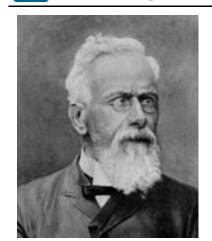
$$d_{\mathsf{GH}}(X,Y) = \frac{1}{2} \min_{\substack{(x_i,y_i) \in \mathcal{C} \\ (x_j,y_j) \in \mathcal{C}}} \max_{\substack{(x_i,y_i) \in \mathcal{C}}} |\delta_X(x_i,x_j) - \delta_Y(y_i,y_j)|$$

$$\forall x_i \exists y_i \text{ s.t.}(x_i, y_i) \in \mathcal{C} \quad \forall y_i \exists x_i \text{ s.t.}(x_i, y_i) \in \mathcal{C}$$

Discrete optimization over correspondences is NP hard!

Distinguish topological spaces





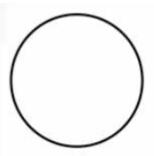
Counts the number of "i-dimensional holes" bi is the "i-th Betti number"



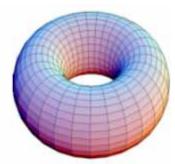
Emmy Noether

(1882-1935)

Enrico Betti (1823-1892)



 $b_1=1$ $b_1 = 0$ $b_2=0$ $b_2=1$



 $b_1 = 2$ $b_2=1$

Betti numbers are computed as dimensions of Boolean vector spaces (E. Noether)

Zomorodian, A. & Carlsson, G. 2005. Computing Persistent Homology. *Discrete &* Computational Geometry, 33, (2), 249-274.

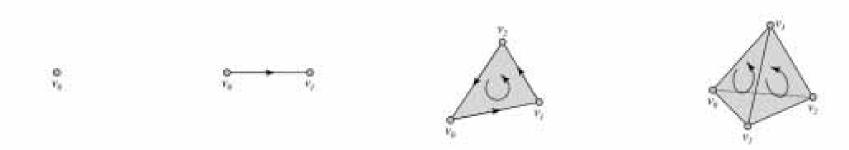




- Statement of Vin de Silva (2003), Pomona College:
- Let M be a topological or metric space, known as the hidden parameter space;
- let \mathbb{R}^d be a Euclidean space, the observation space,
- and let $f: M \to \mathbb{R}^d$ be a continuous embedding.
- Furthermore, let $X \subset M$ be a finite set of data points, perhaps the realization of a stochastic process, i.e., a family of random variables $\{X_i, i \in I\}$ defined on a probability space (Ω, \mathcal{F}, P) , and denote $Y = f(X) \subset \mathbb{R}^d$ the images of these points under the mapping f.
- We refer to X as hidden data, and Y as the observed data.
- M, f and X are unknown, but Y is so can we identify M?







- Mega Problem: To date none of our known methods, algorithms and tools scale to the massive amount and dimensionalities of data we are confronted in practice;
- we need much more research efforts towards making computational topology successful as a general method for data mining and knowledge discovery

Holzinger, A. 2014. On Topological Data Mining. In: Lecture Notes in Computer Science, LNCS 8401. Berlin Heidelberg: Springer, pp. 331-356, doi:10.1007/978-3-662-43968-5_19.

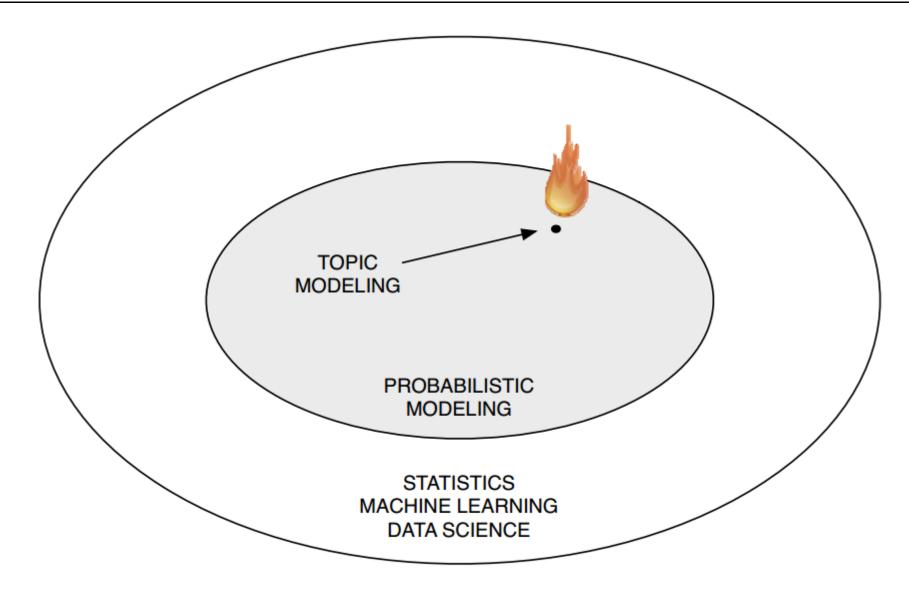




5) Probabilistic Topic Models

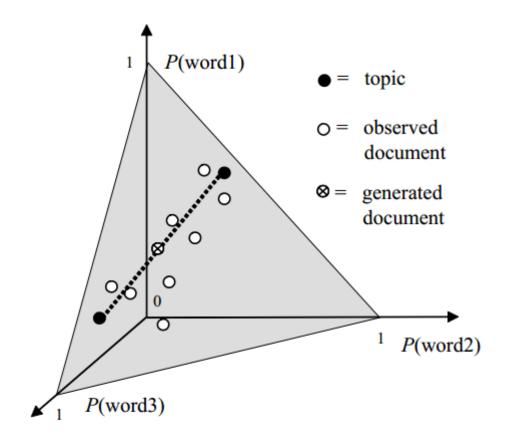
Topic modelling – small topic but hot topic in ML









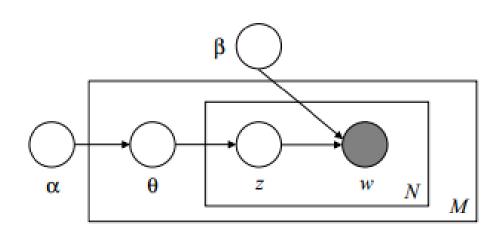


- Documents are categorical distributions over some predefined vocabulary of (10,000+) words
- Topics are categorical distributions on same vocabulary
- Generative model: Each document is (nearly) a convex combination of the topic distributions

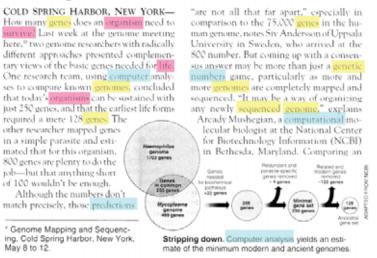
D. Blei, 2008

Generative statistical model for natural language





Seeking Life's Bare (Genetic) Necessities



SCIENCE • VOL. 272 • 24 MAY 1996

Given the parameters α and β , the joint distribution of a topic mixture θ , a set of N topics z, and a set of N words w is given by:

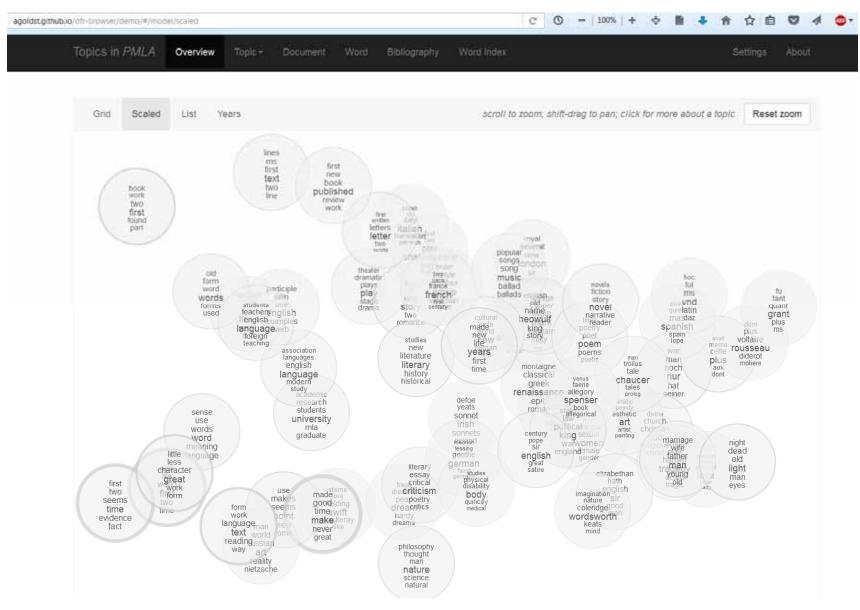
$$p(\theta, \mathbf{z}, \mathbf{w} | \alpha, \beta) = p(\theta | \alpha) \prod_{n=1}^{\infty} p(z_n | \theta) p(w_n | z_n, \beta)$$

Blei, D. M., Ng, A. Y. & Jordan, M. I. 2003. Latent dirichlet allocation. The Journal of machine Learning research, 3, 993-1022.



Motivation: to get insight into unknown document sets

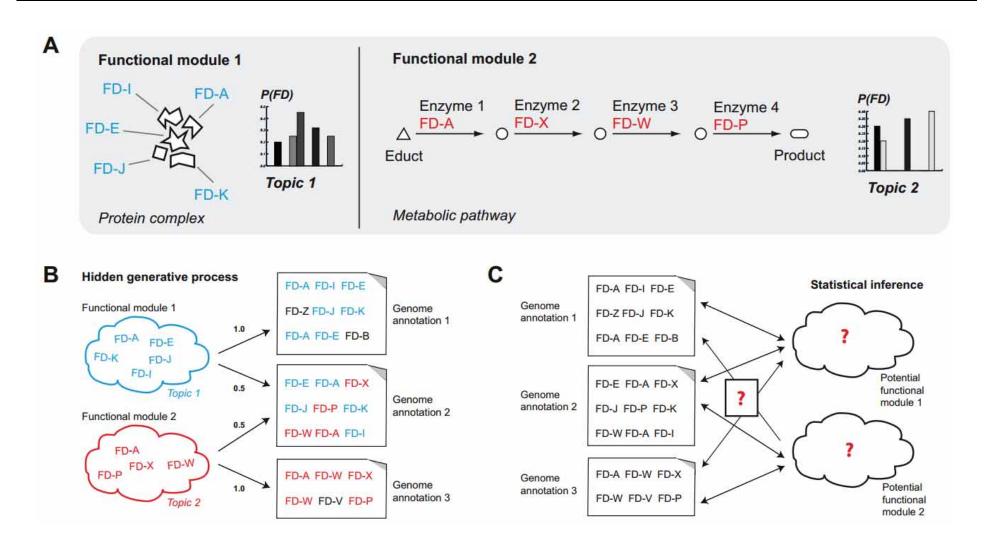




http://agoldst.github.io/dfr-browser/demo/#/model/scaled

Example from Bioinformatics



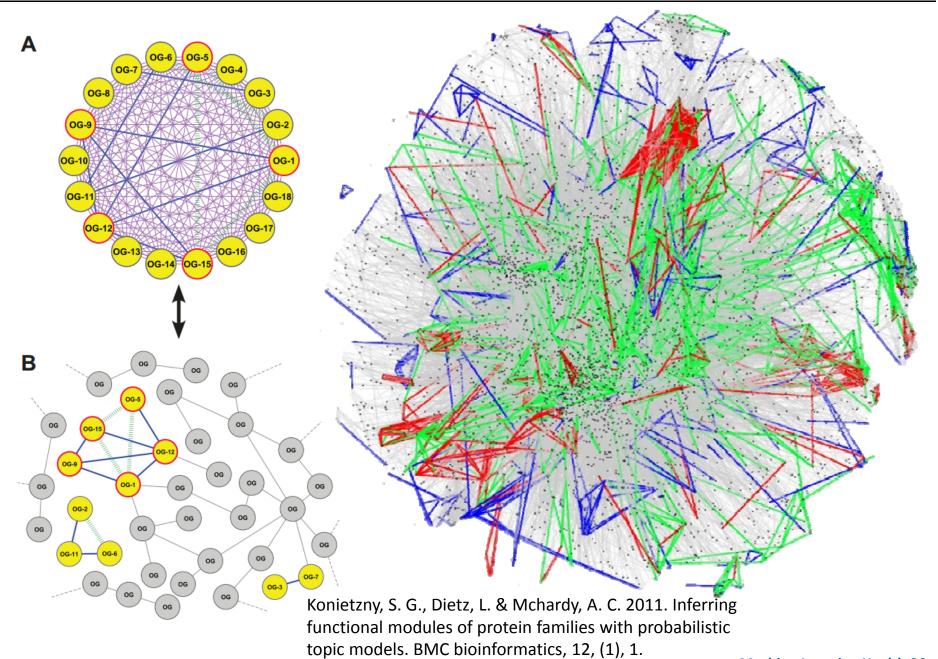


Konietzny, S. G., Dietz, L. & Mchardy, A. C. 2011. Inferring functional modules of protein families with probabilistic topic models. BMC bioinformatics, 12, (1), 1.



Eval. scheme for inferred potential functional modules



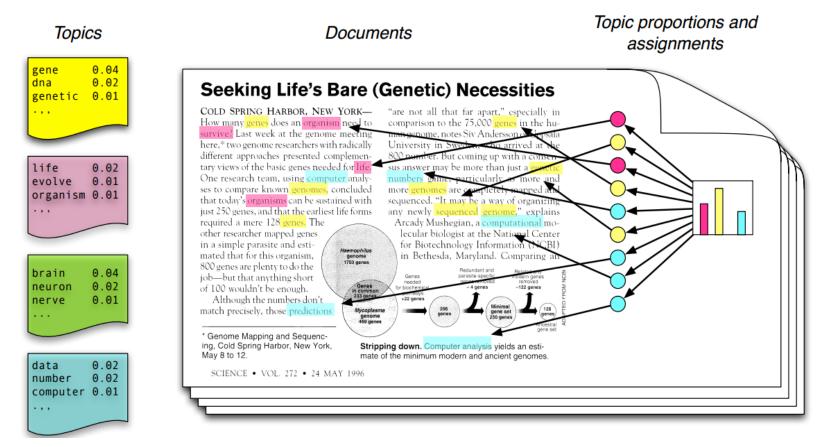






Goal: to get insight in unknown document collections

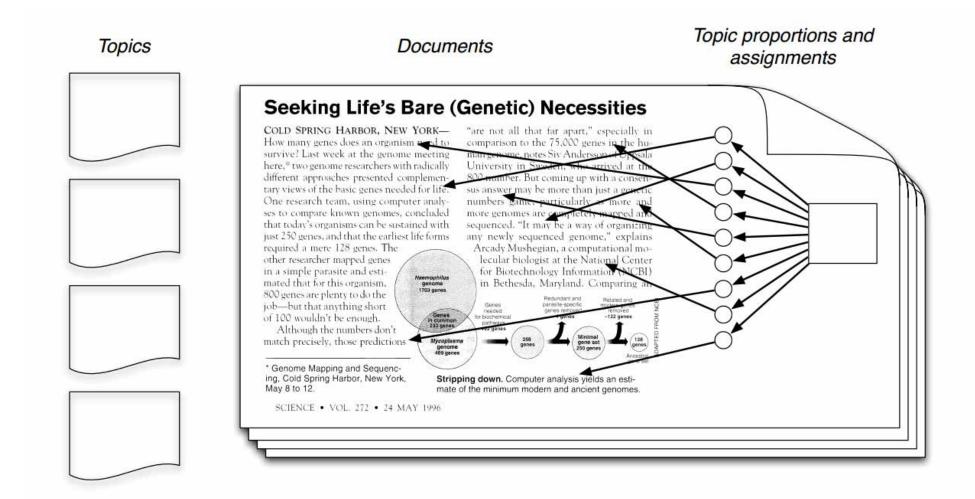
See a nice demo http://agoldst.github.io/dfr-browser/demo/#/model/grid



Each doc is a random mix of corpus-wide topics and each word is drawn from one of these topics







We only observe the docs – the other structure is hidden; then we compute the posterior p(t,p,a|docs)

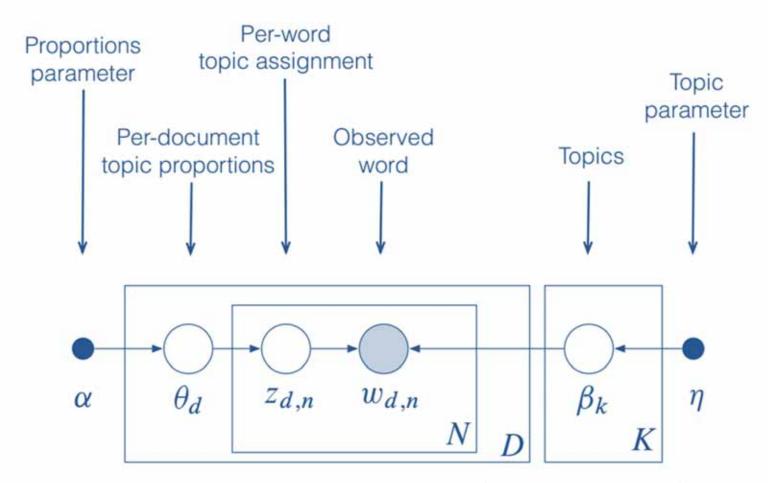
Output Example: 4 learned topics



| human | evolution | disease | computer |
|--------------------------|--------------|-----------------|------------------------|
| genome | evolutionary | host | models |
| dna | species | bacteria | information |
| genetic | organisms | diseases | data |
| genes | life | resistance | computers |
| sequence | origin | bacterial | system |
| gene | biology | new | network |
| molecular | groups | strains | systems |
| sequencing | phylogenetic | control | model |
| $_{\mathrm{map}}$ | living | infectious | parallel |
| information | diversity | $_{ m malaria}$ | methods |
| genetics | group | parasite | networks |
| mapping | new | parasites | software |
| $\operatorname{project}$ | two | united | new |
| sequences | common | tuberculosis | simulations |

LDA is an example for a probabilistic graphical model

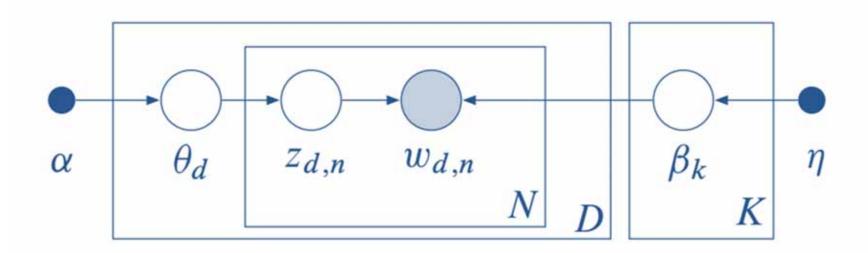




- Encodes assumptions on data with a factorization of the joint
- Connects assumptions to algorithms for computing with data
- Defines the posterior (through the joint)



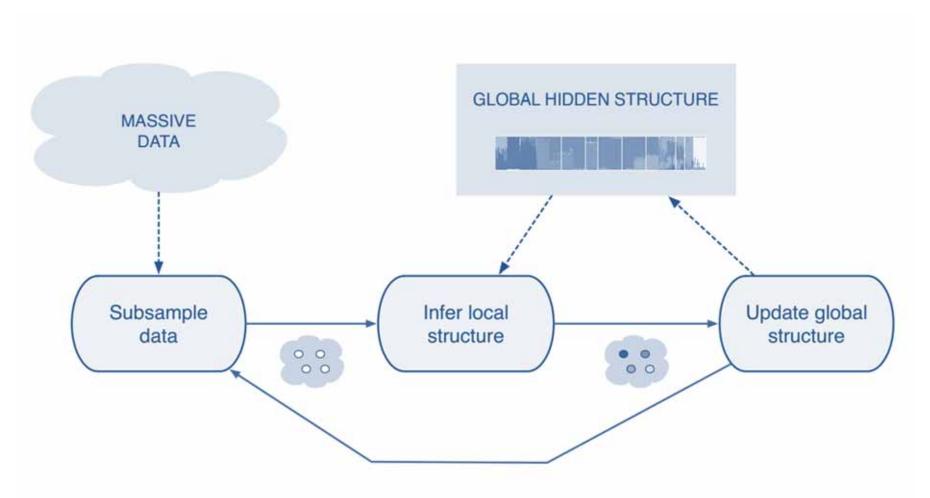




$$p(\beta, \boldsymbol{\theta}, \mathbf{z} | \mathbf{w}) = \frac{p(\beta, \boldsymbol{\theta}, \mathbf{z}, \mathbf{w})}{\int_{\beta} \int_{\boldsymbol{\theta}} \sum_{\mathbf{z}} p(\beta, \boldsymbol{\theta}, \mathbf{z}, \mathbf{w})}$$

We can't compute the denominator, the marginal p(w), therefore we use approximate inference; However, this do not scale well ...





Hoffman, M. D., Blei, D. M., Wang, C. & Paisley, J. 2013. Stochastic variational inference. The Journal of Machine Learning Research, 14, (1), 1303-1347.



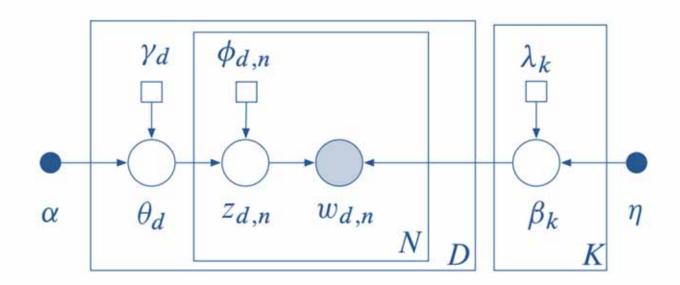


```
1: Initialize \lambda^{(0)} randomly.
 2: Set the step-size schedule \rho_t appropriately.
         Sample a document w_d uniformly from the data set.
       Initialize \gamma_{dk} = 1, for k \in \{1, \dots, K\}.
       repeat
       For n \in \{1, \dots, N\} set
                                \phi_{dn}^k \propto \exp \left\{ \mathbb{E}[\log \theta_{dk}] + \mathbb{E}[\log \beta_{k,w_{dn}}] \right\}, k \in \{1, \dots, K\}.
 8: Set \gamma_d = \alpha + \sum_n \phi_{dn}.
9: until local parameters \phi_{dn} and \gamma_d converge.
10: For k \in \{1, ..., K\} set intermediate topics
                                                       \hat{\lambda}_k = \eta + D \sum_{n=1}^{N} \phi_{dn}^k w_{dn}.
11: Set \lambda^{(t)} = (1 - \rho_t)\lambda^{(t-1)} + \rho_t \hat{\lambda}.
12: until forever
```

Hoffman, M. D., Blei, D. M., Wang, C. & Paisley, J. 2013. Stochastic variational inference. The Journal of Machine Learning Research, 14, (1), 1303-1347.



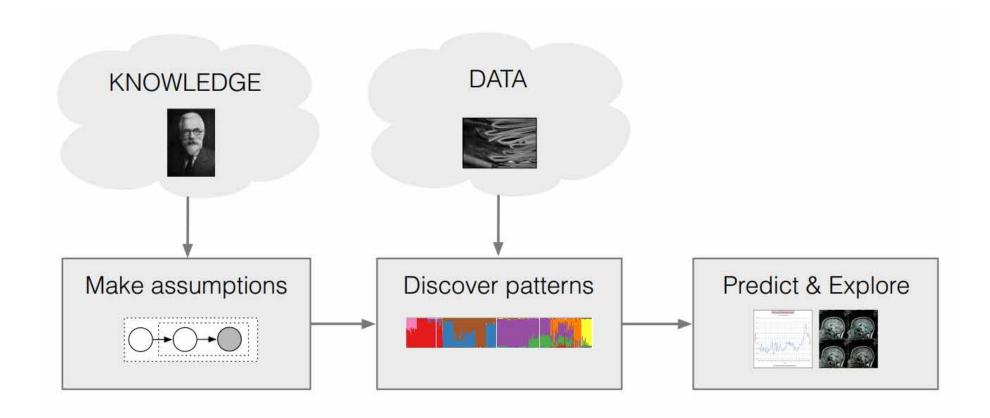




- 1. Sample a document
- 2. Estimate the local variational parameters using the current topics
- 3. Form intermediate topics from those local parameters
- 4. Update topics as a weighted average of intermediate and current topics

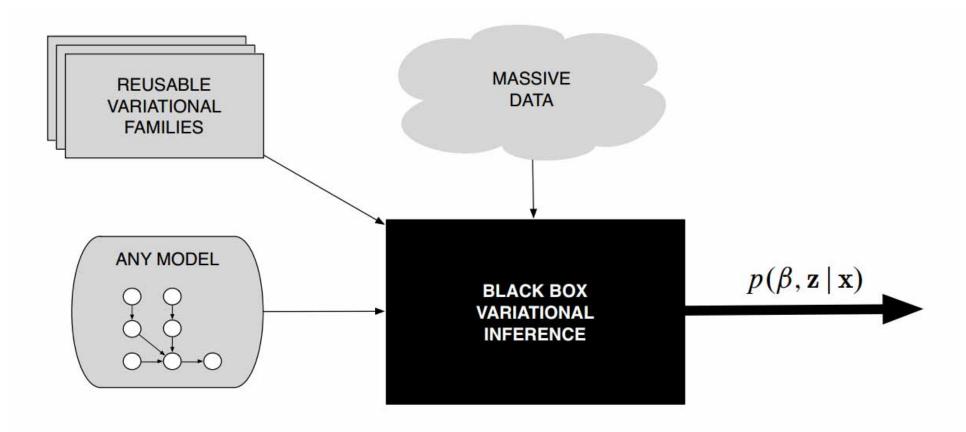
Approximate inference can be difficult to achieve





- Approximate inference can be difficult to derive.
- Especially true for models that are not conditionally conjugate
 (Discrete choice models, Bayesian generalized linear models, ...)
- Holds us back from trying many models.





- Easily use variational inference with any model
- No exponential family requirements
- No mathematical work beyond specifying the model





- Flexible and expressive components for building models
- Scalable and generic inference algorithms
- Easy to use software to stretch probabilistic modeling into new areas
- Topic models are one approach towards detection of topics in text collections
- More general: Identify re-occurring patterns in data collections





Topic model toolkits

- Particular topic models
 - Stanford topic model toolbox http://nlp.stanford.edu/software/tmt
 - ► Topic modeling at Princeton http://www.cs.princeton.edu/~blei/topicmodeling.html
 - ► MALLET (Java) http://mallet.cs.umass.edu
 - Network topic models: Bayes-stack https://github.com/bgamari/bayes-stack
 - ► Gensim (Python) http://radimrehurek.com/gensim/
 - ► R package for Topic models. http://epub.wu.ac.at/3987/
- Frameworks for generative models
 - Variational inference: Infer.net http://research.microsoft.com/infernet/
 - Gibbs sampling: OpenBUGS http://openbugs.net/





6) Graph Bandits

The complexities of optimization: Sébasitien Bubeck



I'm a bandit

Random topics on optimization, probability, and statistics. By Sébastien Bubeck



Home

ORF523: The complexities of optimization

Guest posts

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About me

ORF523: The complexities of optimization

This page collects together the posts for the graduate course on optimization I taught at Princeton in the Spring 2013. This material has been reorganized (some parts have been cut, some have been extended) into a monograph which got recently published "Foundations and Trends in Machine Learning. Vol. 8: No. 3-4. pp 231-357. 2015" (see here for the free version):

https://blogs.princeton.edu/imabandit/



Bubeck, S. & Cesa-Bianchi, N. 2012.
Regret Analysis of
Stochastic and
Nonstochastic Multiarmed Bandit
Problems. Machine
Learning, 5, (1), 1122.

Also very interesting: Bubeck, S. 2015. Convex optimization: Algorithms and complexity. Foundations and Trends in Machine Learning, 8, (3-4), 231-357.

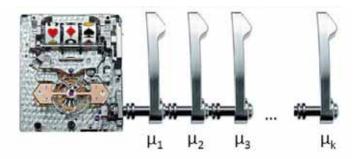




- Slot-machine (bandit robs your money)
- One-armed bandit
- Very simple model for sequential decision making under uncertainty
- Main challenge: exploration versus exploitation
- Many application domains: A/B-Testing, Crowdsourcing, optimization, search, ...



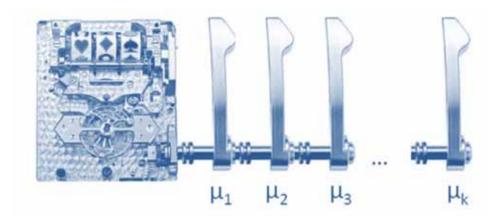




- Multi-armed bandit:= a gambler strategically operating multiple machines in order to draw the highest possible profits
- There are *n* slot-machines ("einarmige Banditen")
- Each machine i returns a reward $y \approx P(y; \Theta_i)$
- Challenge: The machine parameter Θ_i is unknown
- Which arm of a slot machine should a gambler pull to maximize his cumulative reward over a sequence of trials? (stochastic setting or adversarial setting)







Each arm a either wins (reward=1) with fixed (unknown) probability μ_a , or loses (reward=0) with fixed (unknown) probability $1 - \mu_a$

- All draws are independent given $\mu_1 \, \dots \, \mu_k$
- Problem:
 How to pull arms to maximize the total reward?



Underlying Principle of the k-Armed Bandits problem



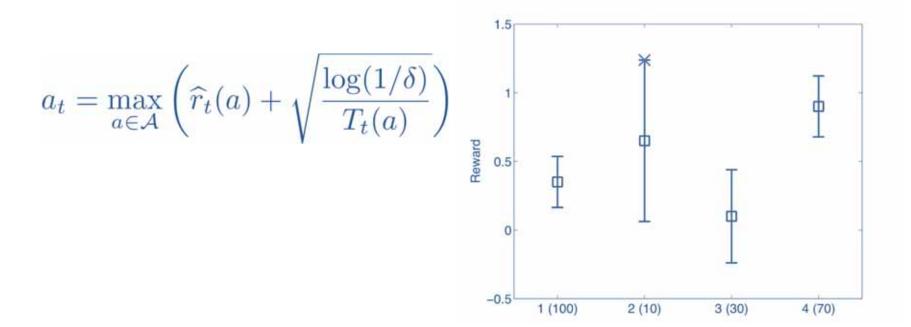
- Let $a_t \in \{1, ..., n\}$ be the choice of a machine at time t
- Let $y_t \in \mathbb{R}$ be the outcome with a mean of $\langle y_{at} \rangle$
- Now, the given policy maps all history to a new choice:

$$\pi: [(a_1, y_1), (a_2, y_2), ..., (a_{t-1}, y_{t-1})] \mapsto a_t$$

- The problem: Find a policy π that $\max \langle y_T \rangle$
- Now, two effects appear when choosing such machine:
 - You collect more data about the machine (=knowledge)
 - You collect reward
- Exploration and Exploitation
 - **Exploration:** Choose the next action a_t to $min\langle H(b_t)\rangle$
 - **Exploitation:** Choose the next action a_t to $max\langle y_t \rangle$
- models an agent that simultaneously attempts to acquire new knowledge (called "exploration") and optimize his or her decisions based on existing knowledge (called "exploitation"). The agent attempts to balance these competing tasks in order to maximize total value over the period of time considered.

More information: http://research.microsoft.com/en-us/projects/bandits





$$a_t = \max_{a \in \mathcal{A}} \left(\text{rew}_t(a) + \text{uncert}_t(a) \right)$$

Exploitation

the higher the (estimated) reward the higher the chance to select the action

Exploration the higher the (theoretical) uncertainty the higher the chance to select the action

Auer, P., Cesa-Bianchi, N. & Fischer, P. 2002. Finite-time analysis of the multiarmed bandit problem. Machine learning, 47, (2-3), 235-256.



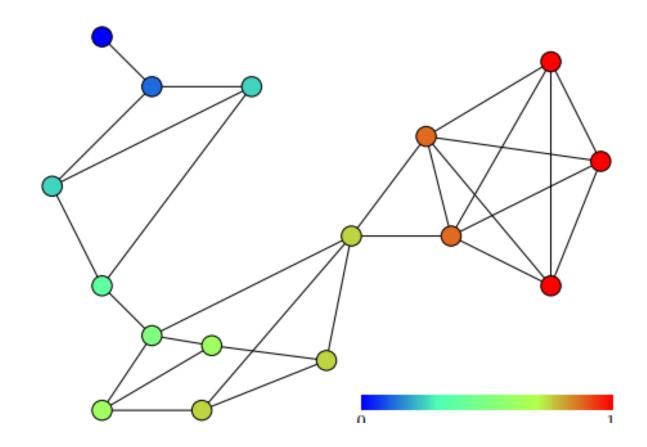
- Let G a known graph with K nodes $\{1, 2, ..., K\}$
- Let f be a unknown function defined on the set of nodes
- For t = 1 to n,
 - Select a node It
 - Observe reward $r_t = f(I_t) + \epsilon_t$
- Goal: maximize sum of expected rewards
- Equivalently minimize regret:

$$R_n = \sum_{t=1}^n (f^* - f(I_t)),$$

where $f^* = \max_{1 \leq i \leq K} f(i)$.

• We care about the case when K > n



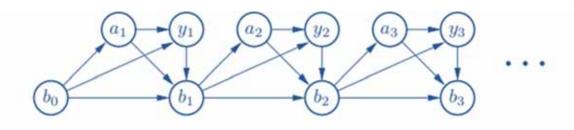




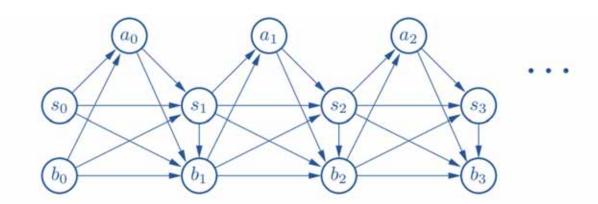


- Knowledge can be represented in two ways:
- 1) as full history $h_t = [(a_1, y_1), (a_2, y_2), ..., (a_{t-1}, y_{t-1})]$ or
- 2) as belief $b_t(\theta) = P(\theta|h_t)$

where Θ are the unknown parameters of all machines. The process can be modelled as belief MDP:



$$P(b'|y,a,b) = \begin{cases} 1 & \text{if } b' = b'_{[b,a,y]} \\ 0 & \text{otherwise} \end{cases}, \quad P(y|a,b) = \int_{\theta_a} b(\theta_a) \; P(y|\theta_a)$$



$$\begin{split} P(b'|s',s,a,b) &= \begin{cases} 1 & \text{if } b' = b[s',s,a] \\ 0 & \text{otherwise} \end{cases}, \quad P(s'|s,a,b) = \int_{\theta} b(\theta) \; P(s'|s,a,\theta) \\ V(b,s) &= \max_{a} \left[\mathsf{E}(r|s,a,b) + \sum_{s'} P(s'|a,s,b) \; V(s',b') \right] \end{split}$$

Poupart, P., Vlassis, N., Hoey, J. & Regan, K. An analytic solution to discrete Bayesian reinforcement learning. Proceedings of the 23rd international conference on Machine learning, 2006. ACM, 697-704.





 Clinical trials: potential treatments for a disease to select from new patients or patient category at each round, see:

W. Thompson. On the likelihood that one unknown probability exceeds another in view of the evidence of two samples. Bulletin of the American Mathematics Society, vol. 25, pp. 285–294, 1933.

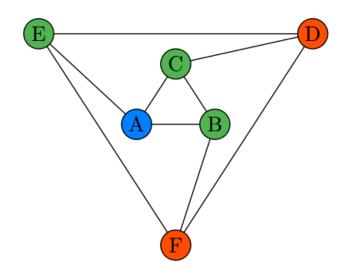
- Games: Different moves at each round, e.g. GO
- Adaptive routing: finding alternative paths, also finding alternative roads for driving from A to B
- Advertisement placements: selection of an ad to display at the Webpage out of a finite set which can vary over time, for each new Web page visitor

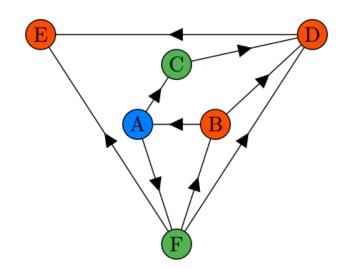














Some Statistics on rare diseases (orphan diseases)



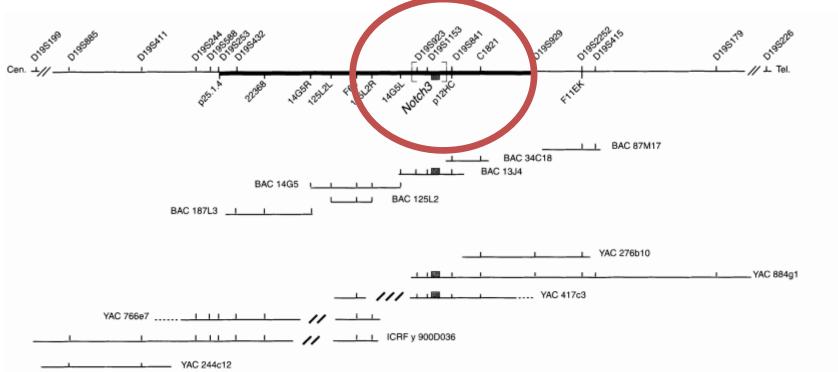
- 7,000 + different types more being discovered every day
- >10% of the world population is suffering (if all of the people with rare diseases lived in one country, it would be the world's 3rd most populous country)
- 80% of rare diseases are genetic, so are present throughout a person's lifetime, even if symptoms do not immediately appear
- >50% of the people affected by rare diseases are children
- Are responsible for 35% of deaths in the first year of life
- The prevalence distribution is skewed 80% of all rare disease patients are affected by 350 rare diseases
- >50% of rare diseases do not have a disease specific foundation supporting or researching their rare disease

https://globalgenes.org/rare-diseases-facts-statistics/ https://www.hon.ch/HONselect/RareDiseases/





- Cerebral Autosomal Dominant Arteriopathy with Subcortical Infarcts and Leukoencephalopathy
- is a hereditary disease affecting all the small cerebral arteries. It causes subcortical infarcts and damages the white matter (leukoencephalopathy) and it is due to various mutations of the Notch3 gene situated on chromosome 19:

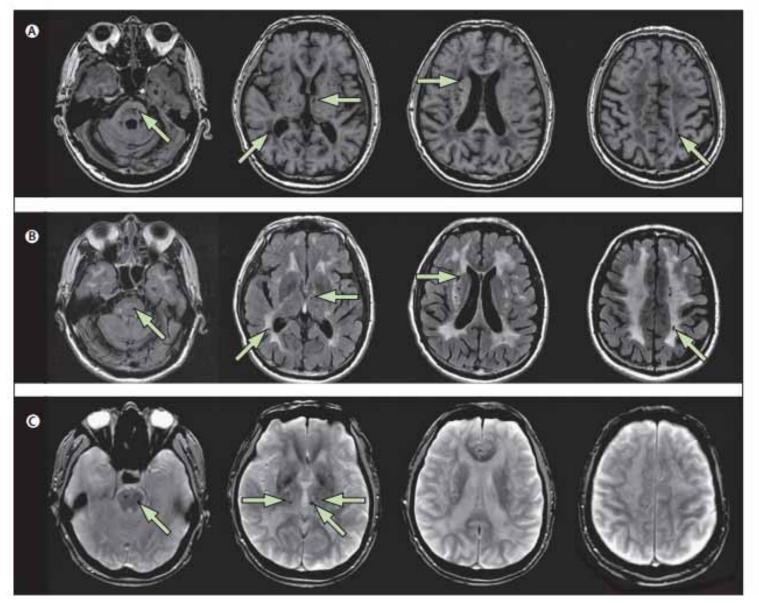


Joutel, A. et al. 1996. Notch3 mutations in CADASIL, a hereditary adult-onset condition causing stroke and dementia. Nature, 383, (6602), 707-710, doi:10.1038/383707a0.

Machine Learning Health 06 Holzinger Group 87







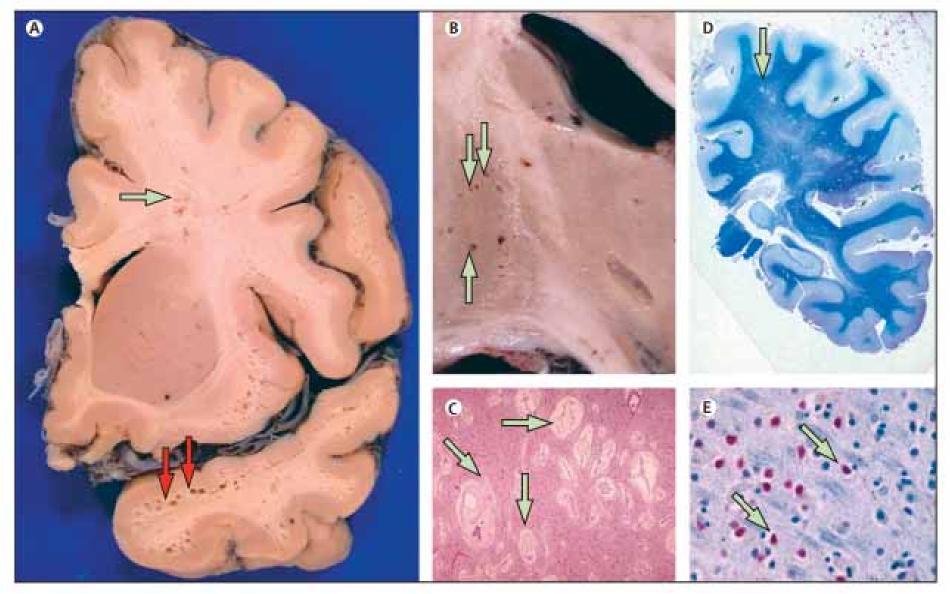
Chabriat, H., Joutel, A., Dichgans, M., Tournier-Lasserve, E. & Bousser, M.-G. 2009. CADASIL. The Lancet Neurology, 8, (7), 643-653, doi:http://dx.doi.org/10.1016/S1474-4422(09)70127-9.

Holzinger Group

Machine Learning Health 06







Chabriat, H., Joutel, A., Dichgans, M., Tournier-Lasserve, E. & Bousser, M.-G. 2009. CADASIL. The Lancet Neurology, 8, (7), 643-653, doi:http://dx.doi.org/10.1016/S1474-4422(09)70127-9.

Holzinger Group

Machine Learning Health 06



Randomized clinical trials have changed little in 70 years, and it's time to revamp the approach by merging clinical research with clinical practice.

In the The Fiddler on the Roof, Tevye sings of tradition: "You may ask, 'how did this tradition get started?' I'll tell you ... [in sotto voce:] I don't know. But it's a tradition!" He could have been describing medical research (Donald A Berry).

http://fortune.com/2015/10/26/cancer-clinical-trial-belmont-report/





Limitations of drug design for rare diseases due to:

- Lack of understanding of the underlying principles of the rare disease
 - Motivation: Research advances
- Unbalanced economic motivation (cost/benefit)
 - Motivation: Orphan Drug Act and other regulations
- Unavailability of # patients for standard trials
 - This is the true bottleneck!

Villar, S. S., Bowden, J. & Wason, J. 2015. Multi-armed Bandit Models for the Optimal Design of Clinical Trials: Benefits and Challenges. 199-215, doi:10.1214/14-STS504.





- The goal of Standard Randomized Controlled Trials (RCT) are a controlled learning setting:
 - Control for Type I and Type II errors, dependent of trial size n_{RCT}
 - In the case if the patient population N is smaller than the trial size n_{RCT} : underpowered trial problem!
- If we change the goal to
- "learning sufficient to treat N as effectively as possible",
- then bandit strategies optimal policy for max. the expected reward - are perfectly suited!

Kuleshov, V. & Precup, D. 2014. Algorithms for multi-armed bandit problems. arXiv:1402.6028.





- Learning → experimenting with all treatments
- Earning → selecting one treatment only, based on experimentation results
- Question 1: How much learning is best for an optimal treatment of N patients?
- Suppose N patients with a rare disease:
 - Experimental Group E and control group C
 - e.g. control = response rate pc and little information about experimental group
- Question 2: How many allocations of treatment to E are necessary (= how much experimentation?)

DYNAMIC PROGRAMMING AND LAGRANGE MULTIPLIERS

By RICHARD BELLMAN

RAND CORPORATION, SANTA MONICA, CALIFORNIA

Communicated by Einar Hille, August 13, 1956

1. Introduction.—The purpose of this note is to indicate how a suitable combination of the classical method of the Lagrange multiplier and the functional-equation method of the theory of dynamic programming¹ can be used to solve numerically, and treat analytically, a variety of variational problems that cannot readily be treated by either method alone.

A series of applications of the method presented here will appear in further publications.

2. Functional Equation Approach.—Consider the problem of maximizing the function

$$F(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} g_i(x_i),$$
 (2.1)

subject to the constraints

(a)
$$\sum_{j=1}^{N} a_{ij}(x_j) \le c_i, \quad i = 1, 2, ..., M,$$

(b) $x_i \ge 0,$ (2.2)



Richard Ernest BELLMAN (1920-1984)

Bellman, R. 1956. Dynamic programming and Lagrange multipliers. Proceedings of the National Academy of Sciences, 42, (10), 767-769.



- According to Whittle (1988) [1], we can now include a third treatment W with $p_W(N)$, so that for a given N it is indifferent between E and W
- If we know pW(N) then the answer to Q2 is:
 Allocate the experimental treatment as long as pw(N) > pc otherwise switch to control group C





Restless Bandits: Activity Allocation in a Changing World

Stable URL: http://www.jstor.org/stable/3214163

P. WHITTLE

Abstract

We consider a population of n projects which in general continue to evolve whether in operation or not (although by different rules). It is desired to choose the projects in operation at each instant of time so as to maximise the expected rate of reward, under a constraint upon the expected number of projects in operation. The Lagrange multiplier associated with this constraint defines an index which reduces to the Gittins index when projects not being operated are static. If one is constrained to operate m projects exactly then arguments are advanced to support the conjecture that, for m and n large in constant ratio, the policy of operating the m projects of largest current index is nearly optimal. The index is evaluated for some particular projects.

GITTINS INDEX; MULTI-ARMED BANDITS; SEQUENTIAL SCHEDULING; STIMULATING PRICES; INDEXABILITY

1. Introduction

The multi-armed bandit problem is a classic version of the problem of optimal allocation of activity under certainty. One can phrase it by saying that one has n projects, the state of project i being denoted by x_i (or by $x_i(t)$ if one wishes to emphasise its dependence on time, t). One can operate only one project at once: if one operates project i then one receives reward $g_i(x_i(t))$ in the time-interval (t, t+1) and the transition $x_i(t) \rightarrow x_i(t+1)$ follows a Markov rule specific to project i. The unused projects neither yield reward nor change state; current states of all projects are known at any time. The problem is to so choose the project at each moment that the expected discounted reward over an infinite future is maximal.

The problem was proposed first during the Second World War, and had





• Start simple: N=1 and $p_E \sim B(1,1)$, what is the value of $p_W(1)$?

$$\max_{E,W}\{1/2, p_W(1)\} \rightarrow p_W(1) = 1/2 = 0.5$$

Then, if $p_C < 0.5$, treatment E is allocated.

• Now, consider N=2 and $p_E\sim B(1,1)$, what is the value of $p_W(2)$?

$$\max_{E,W} \{0.5(1+2/3)+0.5(p_W(2)), 2p_W(2)\}\$$

 $\rightarrow p_W(2) = 5/9 = 0.5556.$

Then, if $p_C < 0.5556$, treatment E is allocated.



• For N=2, suppose that $p_C<0.5556$, so treatment E is allocated to patient 1.

Then, if a success is observed, $p_W(1) = 2/3$ treatment E is allocated to patient 2.

Then, if a failure is observed $p_W(1) = 1/3$ treatment C is allocated to patient 2 if $p_C > 0.333$.

 Working recursively, these "Whittle" indices can be computed for any patient horizon N.





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- Suppose N patients with a rare life threatening disease.
- Two treatments available: control (C) and experimental (E)
- Suppose equipoise so that $p_E, p_C \sim B(1, 1)$
- Q2 How much experimentation? = How many allocations of treatments E and C until choosing one treatment for the rest of the population?





- The $p_W(N)$ can be computed independently for each treatment as the trial evolves.
- If 2 treatments available: C and E, then:

Allocate C to the next patient only if $p_C(N) > p_E(N)$ (Ties broken at random)

• If multiple arms, then the rule is:

Allocate to the next patient the treatment with the highest value of $p_K(N)$.

For the multi-arm case the rule is a near-optimal heuristic.



- Bandit strategy: Is experimentation worth it for a small number N?
- Reconcile clinical trials and clinical practice
- Extensions should deal with randomization, delayed responses and uncertainty around N
- Bayesian bandits need Online-ML
- Bandits are a great source of inspirations and building blocks for solving manifold problems
- Future work: convex optimization, contextual, combinatorial, ...

Berry, D. A. & Fristedt, B. 1985. Bandit problems: sequential allocation of experiments (Monographs on statistics and applied probability), Springer.





Thank you!





- Describe the clinical decision making process!
- Which type of graph is particularly useful for inference and learning?
- What is the key challenge in the application of graphical models for health informatics?
- What was Judea Pearl (1988) discussing in his paper, for which he received the Turing award?
- What main difficulties arise during breast cancer prognosis?
- What can be done to increase the robustness of prognostic cancer tests?
- Inference in Bayes Nets is NP-complete, but there are certain cases where it is tractable, which ones?





- Why do we want to apply ML to graphs?
- Describe typical ML tasks on the example of blood cancer cells!
- If you have a set of points which similarity measures are useful?
- Why is graph comparison in the medical domain useful?
- Why is the Gromov-Hausdorff distance useful?
- What is the central goal of a generative probabilistic model?
- Describe the LDA-model and its application for topic modelling!





- Briefly describe the stochastic variational inference algorithms!
- What is the principle of a bandit?
- How does a multi-armed bandit (MAB) work?
- In which ways can a MAB represent knowledge?
- What is the main problem of a clinical trail and maybe the main problem in clinical medicine?
- Why are rare diseases both important and relevant? Describe an example disease!
- What is the big problem in clinical trials for rare diseases?
- What did Richard Bellman (1956) describe with dynamic programming?
- Why are graph bandits a hot topic for ML research?



Solutions of the Quiz



- 1=this is a factor graph of an undirected graph we have seen this in protein networks (refer to slide Nr. 70 in lecture 5). Factor graph is bipartite and has two types of nodes: Variables, which can be either evidence variables (when we know its value) or query variables (when the value is unknown and we want to predict the value); and factors, which define the relationship between variables in the graph. Each factor can be connected to many variables and comes with a factor function to define the relationship between these variables. For example, if a factor node is connected to two variables nodes A and B, a possible factor function could be imply(A,B), meaning that if the random variable A takes value 1, then so must the random variable B. Each factor function has a weight associated with it, which describes how much influence the factor has on its variables in relative terms. For more information please consult: http://deepdive.stanford.edu/inference
- 2= this is the decomposition of a tree, rooted at nodes into subtrees
- 3= an example for machine translation, Image credit to Kevin Gimpel, Carnegie Mellon University
- 4= the famous expectation-utility theory according to von Neumann and Morgenstern (1954): a decision-maker faced with risky (probabilistic) outcomes of different choices will behave as if he is maximizing the expected value of some function defined over the potential outcomes at some specified point in the future.
- 5= MYCIN –expert system that used early AI (rule-based) to identify bacteria causing severe infections, such as bacteremia and meningitis, and to recommend antibiotics, with the dosage adjusted for patient's body weight the name derived from the antibiotics themselves, as many antibiotics have the suffix "-mycin".
- 6= metabolic and physical processes that determine the physiological and biochemical properties of a cell. These networks comprise the chemical reactions of metabolism, the metabolic pathways, as well as the regulatory interactions that guide these reactions.
- 7= With the sequencing of complete genomes, it is now possible to reconstruct the network of biochemical reactions in many organisms, from bacteria to human. Several of these networks are available online, e.g. Kyoto Encyclopedia of Genes and Genomes (KEGG), EcoCyc, BioCyc etc. Metabolic networks are powerful tools for studying and modelling metabolism.