



Andreas Holzinger  
VO 709.049 Medical Informatics  
30.11.2016 11:15-12:45

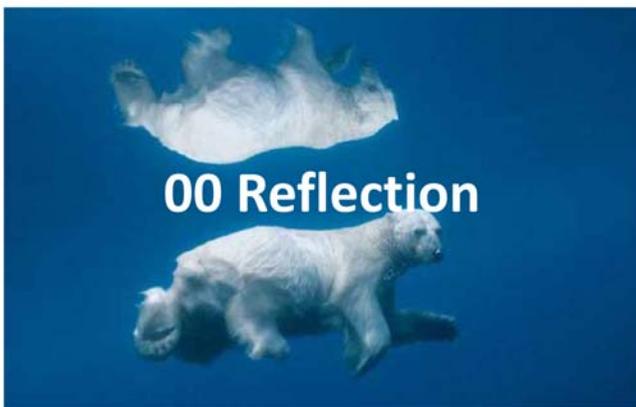


# Lecture 06 Probabilistic Graphical Models II: From Bayesian Networks to Graph Bandits

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<http://hci-kdd.org/biomedical-informatics-big-data>

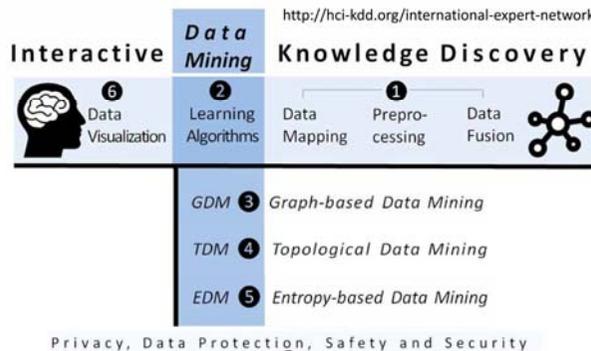


Graph Model

# 01 Graphical Models and Decision Making

Data

$$\mathcal{D} \equiv \{X_1^{(i)}, X_2^{(i)}, \dots, X_m^{(i)}\}_{i=1}^N$$



Holzinger, A. 2014. Trends in Interactive Knowledge Discovery for Personalized Medicine: Cognitive Science meets Machine Learning. IEEE Intelligent Informatics Bulletin, 15, (1), 6-14.

1.  $h_1$  = The identity of ORGANISM 1 is amphibious  
 $h_2$  = PATIENT 1 is female  
 $h_3$  = The name of PATIENT 1 is John Jones

CP( $h_1$ ) = 0.8 There is strongly suggestive evidence (0.8) that the identity of ORGANISM 1 is amphibious  
CP( $h_2$ ) = 0.3 There is weakly suggestive evidence (0.3) that PATIENT 1 is not female  
CP( $h_3$ ) = 0.1 It is unlikely (0.1) that the name of PATIENT 1 is John Jones

2. Protein network diagram

3. Decision tree diagram

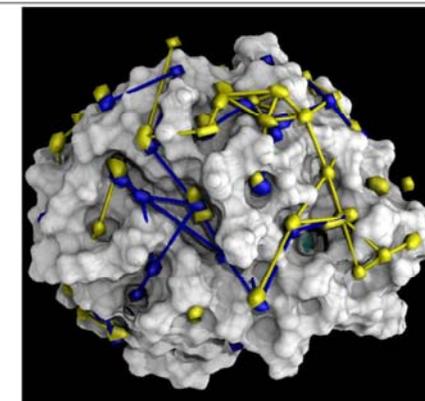
4. Protein network diagram

5. Protein network diagram

6. Protein network diagram

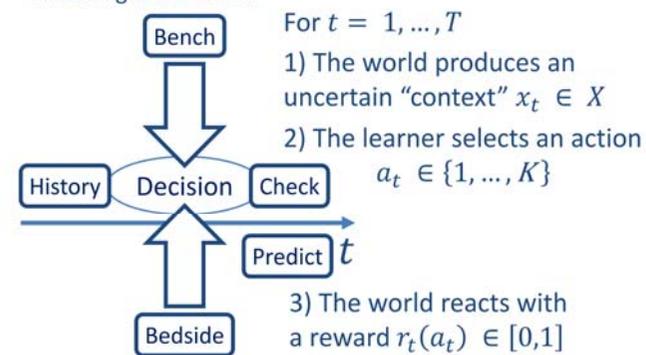


- 00 Reflection – follow-up from last lecture
- 01 Graphical Models and Decision Making
- 02 Bayesian Networks
- 03 Machine Learning on Graphs
- 04 Little Excursus: What is similarity?
- 05 Probabilistic Topic Models
- 06 Graph Bandits (a very hot topic!)

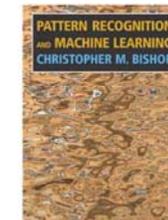


<http://sbcb.bioch.ox.ac.uk/users/oliver/software/>

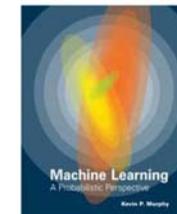
Goal: Learn an **optimal policy** for selecting best actions within a given context



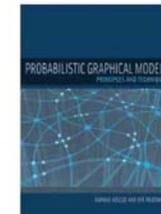
- Medicine is an extremely complex application domain – dealing most of the time with uncertainties -> **probable information!**
- When we have big data but little knowledge automatic ML can help to gain insight:
- Structure learning and prediction in large-scale biomedical networks with probabilistic graphical models**
- If we have little data and deal with NP-hard problems we still need the human-in-the-loop!



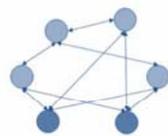
Bishop, C. M. 2007. Pattern Recognition and Machine Learning, Heidelberg, Springer. Chapter 8 on graphical models openly available: <http://research.microsoft.com/en-us/um/people/cmbishop/prml/>



Murphy, K. P. 2012. Machine learning: a probabilistic perspective, MIT press. Chapter 26 (pp. 907) – Graphical model structure learning

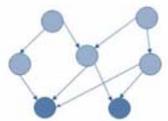


Koller, D. & Friedman, N. 2009. Probabilistic graphical models: principles and techniques, MIT press.



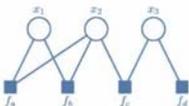
**Undirected:** Markov random fields, useful e.g. for computer vision (Details: Murphy 19)

$$P(\mathbf{X}) = \frac{1}{Z} \exp \left( \sum_{ij} W_{ij} x_i x_j + \sum_i x_i b_i \right)$$



**Directed:** Bayes Nets, useful for designing models (Details: Murphy 10)

$$p(\mathbf{x}) = \prod_{k=1}^K p(x_k | \text{pa}_k)$$



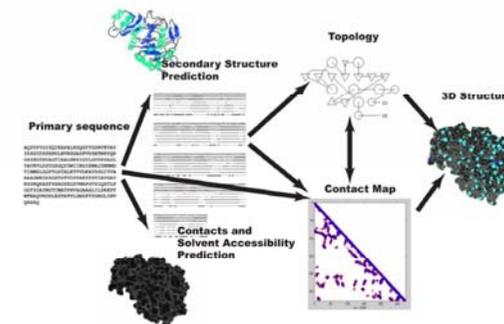
**Factored:** useful for inference/learning

$$p(\mathbf{x}) = \prod_s f_s(\mathbf{x}_s)$$

- What is the advantage of factor graphs?

	Dependency	Efficient Inference	Usage
Bayesian Networks	Yes	Somewhat	Ancestral Generative Process
Markov Networks	Yes	No	Local Couplings and Potentials
Factor Graphs	No	Yes	Efficient, distributed inference

Table credit to Ralf Herbrich, Amazon

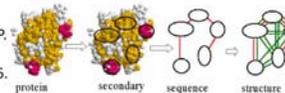


Baldi, P. & Pollastri, G. 2003. The principled design of large-scale recursive neural network architectures--dag-rnns and the protein structure prediction problem. The Journal of Machine Learning Research, 4, 575-602.

- Hypothesis: most biological functions involve the interactions between many proteins, and the complexity of living systems arises as a result of such interactions.
- In this context, the problem of inferring a global protein network for a given organism,
- using all (genomic) data of the organism,
- is one of the main challenges in computational biology

Yamanishi, Y., Vert, J.-P. & Kanehisa, M. 2004. Protein network inference from multiple genomic data: a supervised approach. Bioinformatics, 20, (suppl 1), i363-i370.

Borgwardt, K. M., Ong, C. S., Schönauer, S., Vishwanathan, S., Smola, A. J. & Kriegel, H.-P. 2005. Protein function prediction via graph kernels. Bioinformatics, 21, (suppl 1), i47-i56.



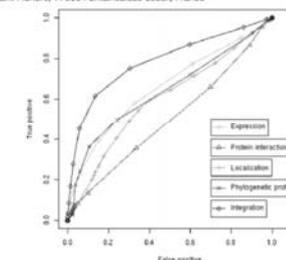
- Important for health informatics: Discovering relationships between biological components
- Unsolved problem in computer science:
- Can the graph isomorphism problem be solved in polynomial time?
  - So far, no polynomial time algorithm is known.
  - It is also not known if it is NP-complete
  - We know that subgraph-isomorphism is NP-complete



**Protein network inference from multiple genomic data: a supervised approach**

Y. Yamanishi<sup>1,\*</sup>, J.-P. Vert<sup>2</sup> and M. Kanehisa<sup>1</sup>

<sup>1</sup>Bioinformatics Center, Institute for Chemical Research, Kyoto University, Gokisocho, Uji, Kyoto 611-0011, Japan and <sup>2</sup>Computational Biology group, Ecole des Mines de Paris, 35 rue Saint-Honoré, 77305 Fontainebleau cedex, France



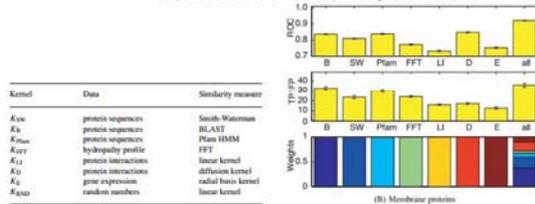
$K_{exp}$  (Expression)  
 $K_{ppi}$  (Protein interaction)  
 $K_{loc}$  (Localization)  
 $K_{phy}$  (Phylogenetic profile)  
 $K_{exp} + K_{ppi} + K_{loc} + K_{phy}$  (Integration)



**A statistical framework for genomic data fusion**

Gert R. G. Lanckriet<sup>1</sup>, Tjil De Bie<sup>2</sup>, Nello Cristianini<sup>3</sup>, Michael I. Jordan<sup>4</sup> and William Stafford Noble<sup>5</sup>\*

<sup>1</sup>Department of Electrical Engineering and Computer Science, <sup>2</sup>Division of Computer Science, Department of Statistics, University of California, Berkeley 94720, USA, <sup>3</sup>Department of Electrical Engineering, ESAT-SCD, Katholieke Universiteit Leuven 3001, Belgium, <sup>4</sup>Department of Statistics, University of California, Davis 95616, USA and <sup>5</sup>Department of Genome Sciences, University of Washington, Seattle 98195, USA



Lanckriet, G. R., De Bie, T., Cristianini, N., Jordan, M. I. & Noble, W. S. 2004. A statistical framework for genomic data fusion. *Bioinformatics*, 20, (16), 2626-2635.

# 02 Bayesian Networks "Bayes' Nets"

- is a **probabilistic model**, consisting of two parts:
  - 1) a dependency structure and
  - 2) local probability models.

$$p(x_1, \dots, x_n) = \prod_{i=1}^n p(x_i | Pa(x_i))$$

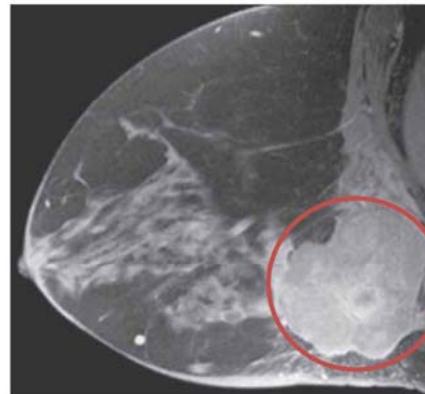
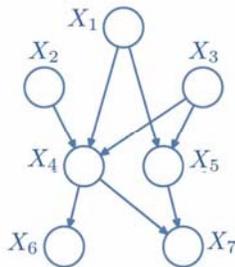
Where  $Pa(x_i)$  are the parents of  $x_i$

BN inherently model the **uncertainty in the data**. They are a successful marriage between probability theory and graph theory; allow to model a multidimensional probability distribution in a sparse way by searching independency relations in the data. Furthermore this model allows different strategies to integrate two data sources.

Pearl, J. (1988) *Probabilistic reasoning in intelligent systems: networks of plausible inference*. San Francisco, Morgan Kaufmann.

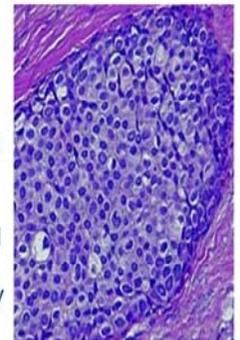
$$p(X_1, \dots, X_7) =$$

$$p(X_1)p(X_2)p(X_3)p(X_4|X_1, X_2, X_3) \cdot p(X_5|X_1, X_3)p(X_6|X_4)p(X_7|X_4, X_5)$$

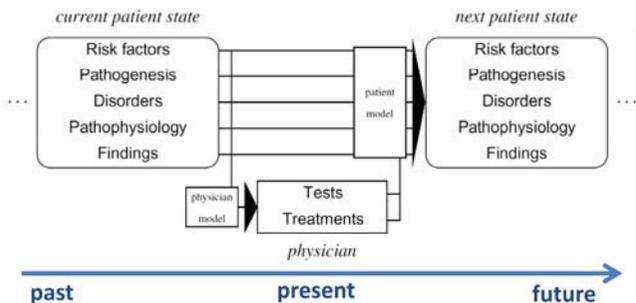


Overmoyer, B. A., Lee, J. M. & Lerwill, M. F. (2011) Case 17-2011 A 49-Year-Old Woman with a Mass in the Breast and Overlying Skin Changes. *New England Journal of Medicine*, 364, 23, 2246-2254.

- = the prediction of the future course of a disease conditional on the patient's history and a projected treatment strategy
- Danger: probable Information !
- Therefore valid prognostic models can be of great benefit for clinical decision making and of great value to the patient, e.g., for notification and quality of-life decisions



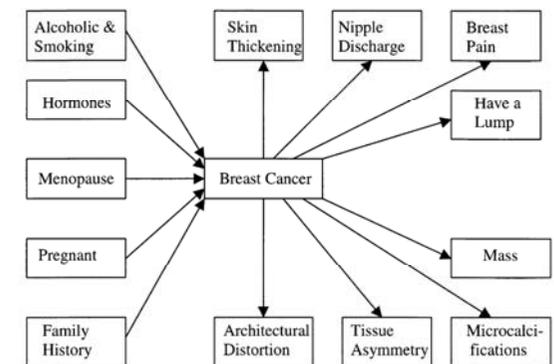
Knaus, W. A., Wagner, D. P. & Lynn, J. (1991) Short-term mortality predictions for critically ill hospitalized adults: science and ethics. *Science*, 254, 5030, 389.



van Gerven, M. A. J., Taal, B. G. & Lucas, P. J. F. (2008) Dynamic Bayesian networks as prognostic models for clinical patient management. *Journal of Biomedical Informatics*, 41, 4, 515-529.

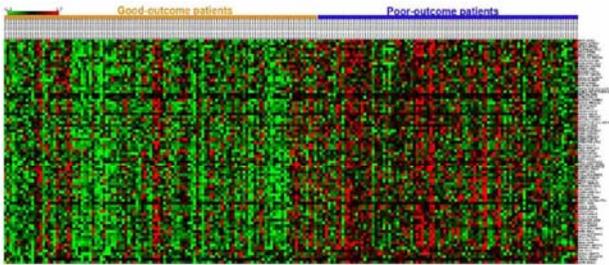
Category	Node description	State description
Diagnosis	Breast cancer	Present, absent.
Clinical history	Habit of drinking alcoholic beverages and smoking	Yes, no.
	Taking female hormones	Yes, no.
	Have gone through menopause	Yes, no.
	Have ever been pregnant	Yes, no.
	Family member has breast cancer	Yes, no.
Physical findings	Nipple discharge	Yes, no.
	Skin thickening	Yes, no.
	Breast pain	Yes, no.
Mammographic findings	Have a lump(s)	Yes, no.
	Architectural distortion	Present, absent.
	Mass	Score from one to three, score from four to five, absent
	Microcalcification cluster	Score from one to three, score from four to five, absent
	Asymmetry	Present, absent.

Wang, X. H., et al. (1999) Computer-assisted diagnosis of breast cancer using a data-driven Bayesian belief network. *International Journal of Medical Informatics*, 54, 2, 115-126.



Wang, X. H., et al. (1999) Computer-assisted diagnosis of breast cancer using a data-driven Bayesian belief network. *International Journal of Medical Informatics*, 54, 2, 115-126.

- Integrating microarray data from multiple studies to increase sample size;
- = approach to the development of more robust prognostic tests

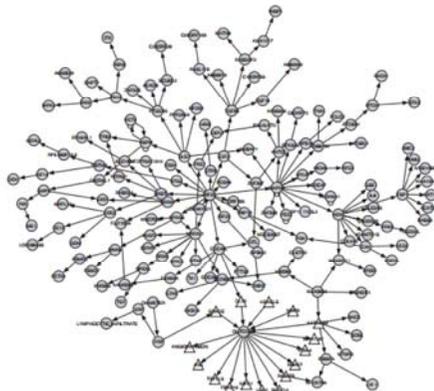


Xu, L., Tan, A., Winslow, R. & Geman, D. (2008) Merging microarray data from separate breast cancer studies provides a robust prognostic test. *BMC Bioinformatics*, 9, 1, 125-139.

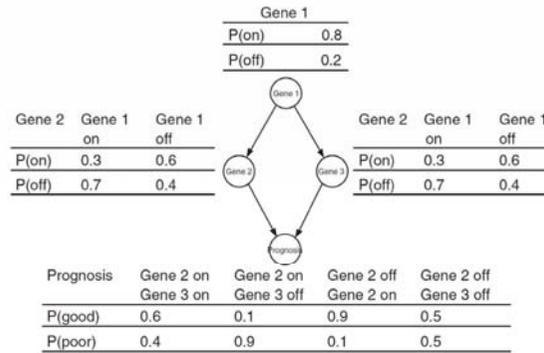
- First the structure is learned using a search strategy.
- Since the number of possible structures increases super exponentially with the number of variables,
- the well-known greedy search algorithm K2 can be used in combination with the Bayesian Dirichlet (BD) scoring metric:

$$p(S|D) \propto p(S) \prod_{i=1}^n \prod_{j=1}^{q_i} \left[ \frac{\Gamma(N'_{ij})}{\Gamma(N'_{ij} + N_{ij})} \prod_{k=1}^{r_i} \frac{\Gamma(N'_{ijk} + N_{ijk})}{\Gamma(N'_{ijk})} \right]$$

$N_{ijk}$  ... number of cases in the data set  $D$  having variable  $i$  in state  $k$  associated with the  $j$ -th instantiation of its parents in current structure  $S$ .  
 $n$  is the total number of variables.



Gevaert, O., Smet, F. D., Timmerman, D., Moreau, Y. & Moor, B. D. (2006) Predicting the prognosis of breast cancer by integrating clinical and microarray data with Bayesian networks. *Bioinformatics*, 22, 14, 184-190.

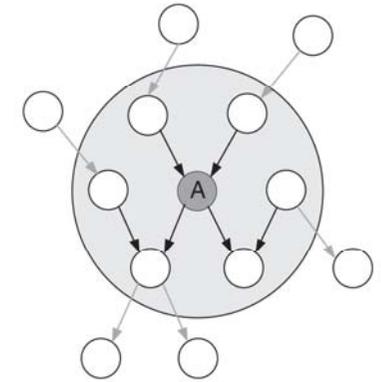


Gevaert, O., Smet, F. D., Timmerman, D., Moreau, Y. & Moor, B. D. (2006) Predicting the prognosis of breast cancer by integrating clinical and microarray data with Bayesian networks. *Bioinformatics*, 22, 14, 184-190.

- Next,  $N_{ij}$  is calculated by summing over all states of a variable:
- $N_{ij} = \sum_{k=1}^{r_i} N_{ijk}$  and  $N'_{ij}$  have similar meanings but refer to prior knowledge for the parameters.
- When no knowledge is available they are estimated using  $N_{ijk} = N / (r_i q_i)$
- with  $N$  the equivalent sample size,
- $r_i$  the number of states of variable  $i$  and
- $q_i$  the number of instantiations of the parents of variable  $i$ .
- $\Gamma(\cdot)$  corresponds to the gamma distribution.
- Finally  $p(S)$  is the prior probability of the structure.
- $p(S)$  is calculated by:
- $p(S) = \prod_{i=1}^n \prod_{l=1}^{p_i} p(l_i \rightarrow x_i) \prod_{m=1}^{o_i} p(m_i x_i)$
- with  $p_i$  the number of parents of variable  $x_i$  and  $o_i$  all the variables that are not a parent of  $x_i$ .
- Next,  $p(a \rightarrow b)$  is the probability that there is an edge from  $a$  to  $b$  while  $p(ab)$  is the inverse, i.e. the probability that there is no edge from  $a$  to  $b$

- For certain cases it is tractable if:
  - Just one variable is unobserved
  - We have singly connected graphs (no undirected loops -> belief propagation)
  - Assigning probability to fully observed set of variables
- Possibility: Monte Carlo Methods (generate many samples according to the Bayes Net distribution and then count the results)
- Otherwise: approximate solutions, NOTE:
 

**Sometimes it is better to have an approximate solution to a complex problem – than a perfect solution to a simplified problem**



Gevaert, O., Smet, F. D., Timmerman, D., Moreau, Y. & Moor, B. D. (2006) Predicting the prognosis of breast cancer by integrating clinical and microarray data with Bayesian networks. *Bioinformatics*, 22, 14, 184-190.

- Estimating the parameters of the local probability models corresponding with the dependency structure.
- CPTs are used to model these local probability models.
- For each variable and instantiation of its parents there exists a CPT that consists of a set of parameters.
- Each set of parameters was given a uniform Dirichlet prior:

$$p(\theta_{ij}|S) = Dir(\theta_{ij}|N'_{ij1}, \dots, N'_{ijk}, \dots, N'_{ijr_i})$$

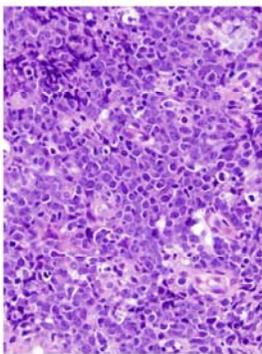
Note: With  $\theta_{ij}$  a parameter set where  $i$  refers to the variable and  $j$  to the  $j$ -th instantiation of the parents in the current structure.  $\theta_{ij}$  contains a probability for every value of the variable  $x_i$  given the current instantiation of the parents.  $Dir$  corresponds to the Dirichlet distribution with  $(N'_{ij1}, \dots, N'_{ijr_i})$  as parameters of this Dirichlet distribution. Parameter learning then consists of updating these Dirichlet priors with data. This is straightforward because the multinomial distribution that is used to model the data, and the Dirichlet distribution that models the prior, are conjugate distributions. This results in a Dirichlet posterior over the parameter set:

$$p(\theta_{ij}|D, S) = Dir(\theta_{ij}|N'_{ij1} + N_{ij1}, \dots, N'_{ijk} + N_{ijk}, \dots, N'_{ijr_i} + N_{ijr_i})$$

with  $N_{ijk}$  defined as before.

# 3) Machine Learning on Graphs

The two main forms of lymphoma are Hodgkin lymphoma and non-Hodgkin lymphoma (NHL). Lymphoma occurs when cells of the immune system called lymphocytes, a type of white blood cell, grow and multiply uncontrollably. Cancerous lymphocytes can travel to many parts of the body, including the lymph nodes, spleen, bone marrow, blood, or other organs, and form a mass called a tumor. The body has two main types of lymphocytes that can develop into lymphomas: B-lymphocytes (B-cells) and T-lymphocytes (T-cells).



www.lymphoma.org

http://imagebank.hematology.org/

Nodes: proteins  
Links: physical interactions (binding)  
Puzzling pattern:  
Hubs tend to link to small degree nodes.

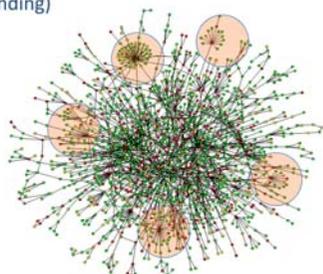
Why is this puzzling?

In a random network, the probability that a node with degree  $k$  links to a node with degree  $k'$  is:

$$p_{kk'} = \frac{kk'}{2L}$$

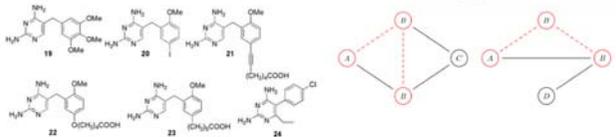
$$k=50, k'=13, N=1,458, L=1746$$

$$p_{50,13} = 0.15 \quad p_{2,3} = 0.0004$$



Jeong, H., Mason, S. P., Barabasi, A. L. & Oltvai, Z. N. 2001. Lethality and centrality in protein networks. Nature, 411, (6833), 41-42.

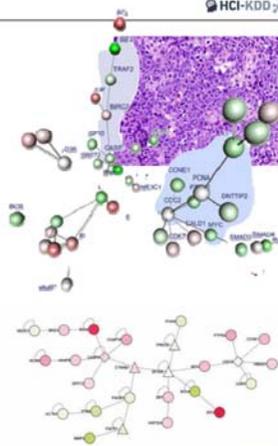
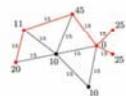
Sequence alignment of B. cereus, B. anthracis, B. colli, and B. spizizenii proteins. The alignment shows conserved regions across the different species.



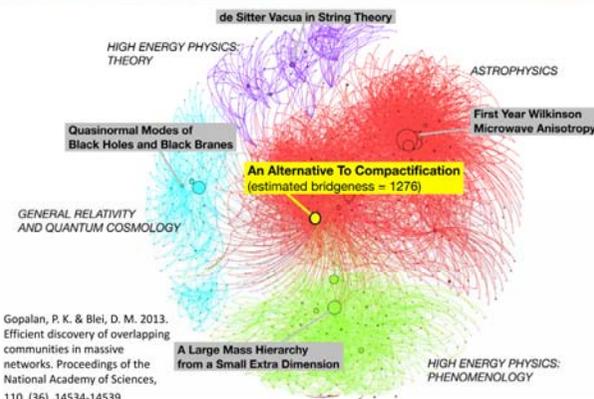
How similar are two graphs? How similar is their structure? How similar are their node and edge labels?

Joska, T. M. & Anderson, A. C. 2006. Structure-activity relationships of Bacillus cereus and Bacillus anthracis dihydrofolate reductase: toward the identification of new potent drug leads. Antimicrobial agents and chemotherapy, 50, 3435-3443.

- Discover unexplored interactions in PPI-networks and gene regulatory networks
- Learn the structure
- Reconstruct the structure

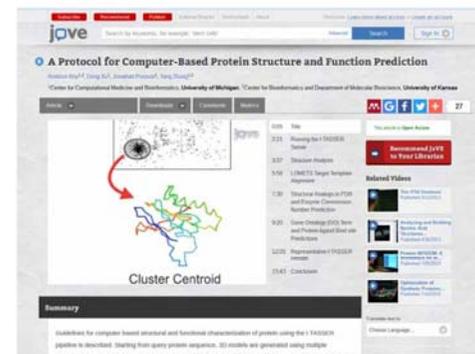


Dittrich, M. T., Klau, G. W., Rosenwald, A., Dandekar, T. & Müller, T. 2008. Identifying functional modules in protein-protein interaction networks: an integrated exact approach. Bioinformatics, 24, (13), 1223-1231.



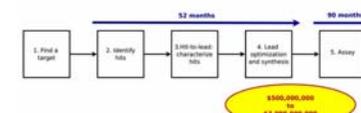
Gopalan, P. K. & Blei, D. M. 2013. Efficient discovery of overlapping communities in massive networks. Proceedings of the National Academy of Sciences, 110, (36), 14534-14539.

- Similar Property Principle: Molecules having similar structures should have similar activities.
- Structure-based representations: Compare molecules by comparing substructures, e.g.
  - Sets as vectors: Measure similarity by the cosine distance
  - Sets as sets: Measure similarity by the Jaccard distance
  - Sets as points: Measure similarity by Euclidean distance
- Problems: Dimensionality, Non-Euclidean cases



http://www.jove.com/video/3259/a-protocol-for-computer-based-protein-structure-function

- A) Discovery of unexplored interactions
- B) Learning and Predicting the structure
- C) Reconstructing the structure
- Which joint probability distributions does a graphical model represent?
- How can we learn the parameters and structure of a graphical model?



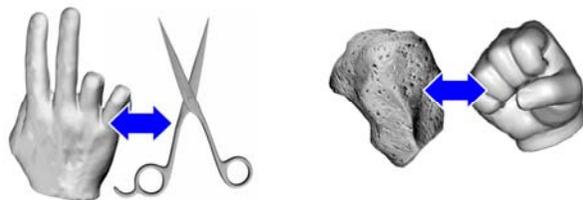
The chemical space

- $10^{60}$  possible small organic molecules
- $10^{22}$  stars in the observable universe

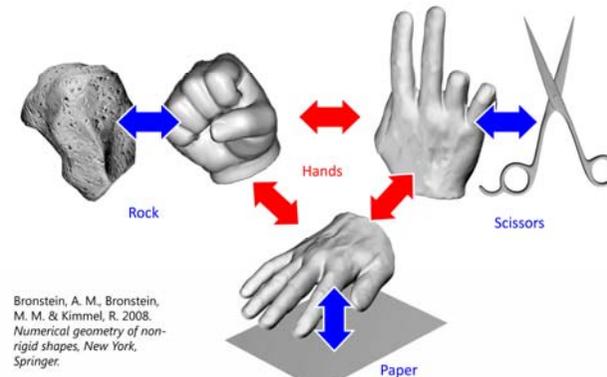
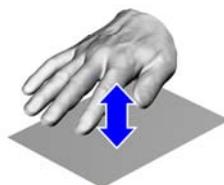
# 4) Little Excursus: What is similarity?



Image credit to Eamonn Keogh (2008)



Bronstein, A. M., Bronstein, M. M. & Kimmel, R. 2008. Numerical geometry of non-rigid shapes, New York, Springer.

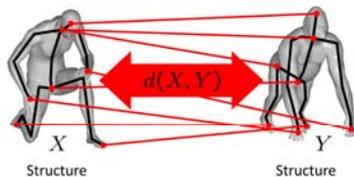


Bronstein, A. M., Bronstein, M. M. & Kimmel, R. 2008. Numerical geometry of non-rigid shapes, New York, Springer.

Similarity and Correspondence

Bronstein, A. M., Bronstein, M. M. & Kimmel, R. 2008. Numerical geometry of non-rigid shapes, New York, Springer.

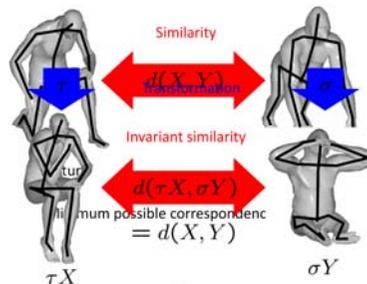
<http://www.inf.usi.ch/bronstein/>



Correspondence quality = structure similarity (distortion)

Minimum possible correspondence distortion

Invariant Similarity



Gromov-Hausdorff dist: finding the opt. correspondence

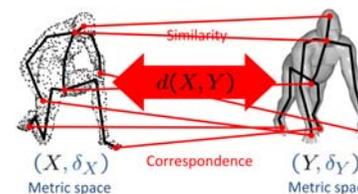


Michael Gromov (1943-)

Gromov, M. (1984) Infinite groups as geometric objects.



Felix Hausdorff (1868-1942)

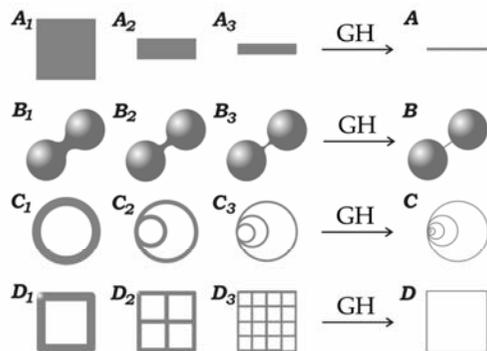


$$d_{GH}(X, Y) = \frac{1}{2} \min_C \max_{(x_i, y_i) \in C} |\delta_X(x_i, x_j) - \delta_Y(y_i, y_j)|$$

$$\forall x_i \exists y_i \text{ s.t. } (x_i, y_i) \in C \quad \forall y_i \exists x_i \text{ s.t. } (x_i, y_i) \in C$$

Discrete optimization over correspondences is NP hard !

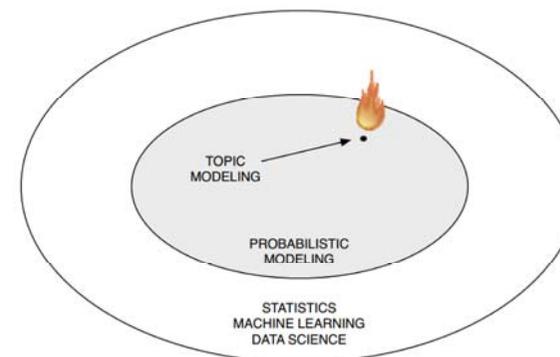
Example

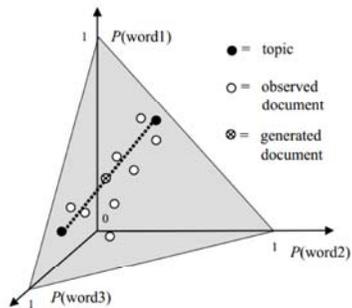


Sormani, C. 2010. How Riemannian Manifolds Converge: A Survey. arXiv preprint arXiv:1006.0411.

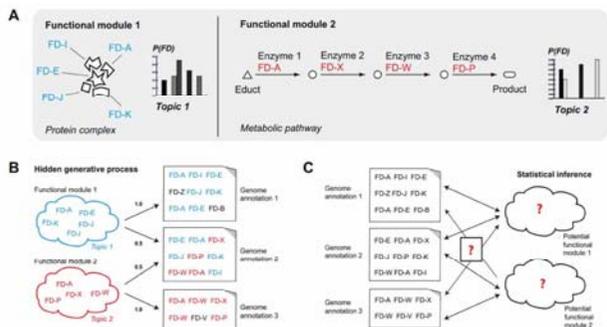
# 5) Probabilistic Topic Models

Topic modelling – small topic but hot topic in ML





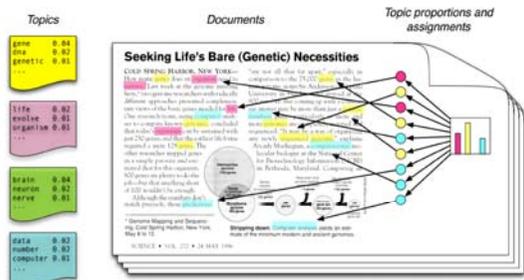
Blei, D. M. 2012. Probabilistic topic models. Communications of the ACM, 55, (4), 77-84, doi:10.1145/2133806.2133826.



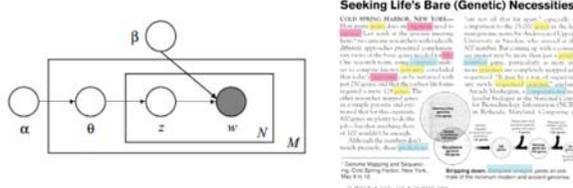
Konietzny, S. G., Dietz, L. & Mchardy, A. C. 2011. Inferring functional modules of protein families with probabilistic topic models. BMC bioinformatics, 12, (1), 1.

Goal: to get insight in unknown document collections

See a nice demo <http://agoldst.github.io/dfp-browser/demo/#/model/grid>



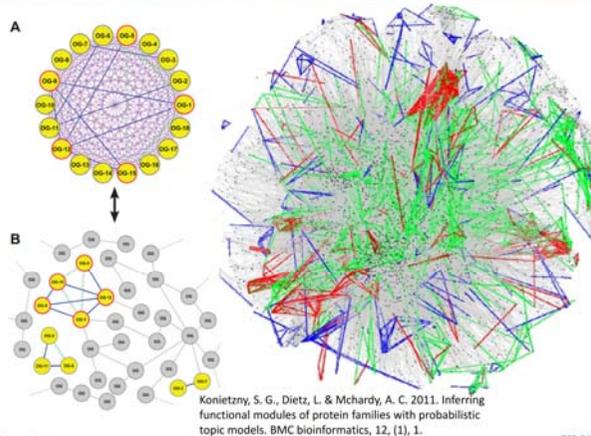
Each doc is a random mix of corpus-wide topics and each word is drawn from one of these topics



Given the parameters  $\alpha$  and  $\beta$ , the joint distribution of a topic mixture  $\theta$ , a set of  $N$  topics  $z$ , and a set of  $N$  words  $w$  is given by:

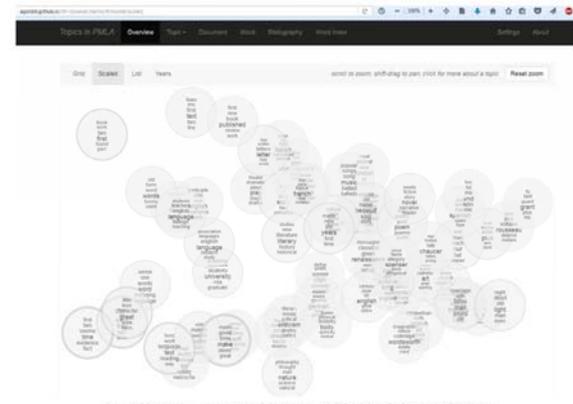
$$p(\theta, z, w | \alpha, \beta) = p(\theta | \alpha) \prod_{n=1}^N p(z_n | \theta) p(w_n | z_n, \beta)$$

Blei, D. M., Ng, A. Y. & Jordan, M. I. 2003. Latent dirichlet allocation. The Journal of machine Learning research JMLR, 3, 993-1022.

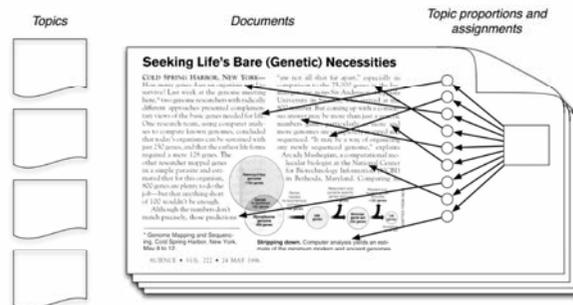


Konietzny, S. G., Dietz, L. & Mchardy, A. C. 2011. Inferring functional modules of protein families with probabilistic topic models. BMC bioinformatics, 12, (1), 1.

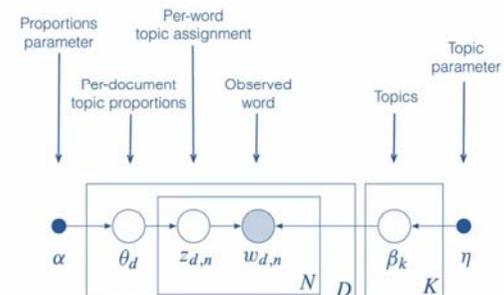
human	evolution	disease	computer
genome	evolutionary	host	models
dna	species	bacteria	information
genetic	organisms	diseases	data
genes	life	resistance	computers
sequence	origin	bacterial	system
gene	biology	new	network
molecular	groups	strains	systems
sequencing	phylogenetic	control	model
map	living	infectious	parallel
information	diversity	malaria	methods
genetics	group	parasite	networks
mapping	new	parasites	software
project	two	united	new
sequences	common	tuberculosis	simulations



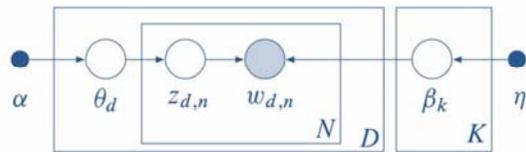
<http://agoldst.github.io/dfp-browser/demo/#/model/scaled>



We only observe the docs – the other structure is hidden; then we compute the posterior  $p(\theta, p, \alpha | docs)$



- Encodes assumptions on data with a factorization of the joint
- Connects assumptions to algorithms for computing with data
- Defines the posterior (through the joint)



$$p(\beta, \theta, z | w) = \frac{p(\beta, \theta, z, w)}{\int_{\beta} \int_{\theta} \sum_z p(\beta, \theta, z, w)}$$

We can't compute the denominator, the marginal  $p(w)$ , therefore we use approximate inference; However, this do not scale well ...

```

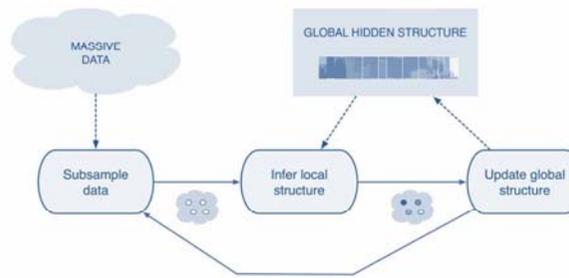
1: Initialize  $\lambda^{(0)}$  randomly.
2: Set the step-size schedule  $\rho_t$  appropriately.
3: repeat
4:   Sample a document  $w_d$  uniformly from the data set.
5:   Initialize  $\gamma_{dk} = 1$ , for  $k \in \{1, \dots, K\}$ .
6:   repeat
7:     For  $n \in \{1, \dots, N\}$  set
       
$$\phi_{dn}^k \propto \exp\{\mathbb{E}[\log \theta_{dn}] + \mathbb{E}[\log \beta_{k, w_{dn}}]\}, k \in \{1, \dots, K\}.$$

8:     Set  $\gamma_d = \alpha + \sum_n \phi_{dn}$ .
9:     until local parameters  $\phi_{dn}^k$  and  $\gamma_d$  converge.
10:    For  $k \in \{1, \dots, K\}$  set intermediate topics
       
$$\tilde{\lambda}_k = \eta + D \sum_{n=1}^N \phi_{dn}^k w_{dn}.$$

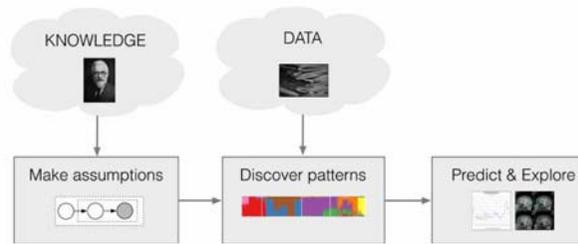
11:    Set  $\lambda^{(t)} = (1 - \rho_t) \lambda^{(t-1)} + \rho_t \tilde{\lambda}$ .
12:   until forever
    
```

Hoffman, M. D., Blei, D. M., Wang, C. & Paisley, J. 2013. Stochastic variational inference. The Journal of Machine Learning Research, 14, (1), 1303-1347.

- **Flexible** and expressive components for building models are of utmost importance
- **Scalable** and generic inference algorithms (multi-task and transfer learning)
- **Usability** gets a totally new importance: Easy to use algorithms for the non-expert user to stretch probabilistic modeling into new areas
- Topic models are **one** approach towards detection of topics in document collections
- Example: Identifying re-occurring patterns in such data collections (gaining new knowledge)

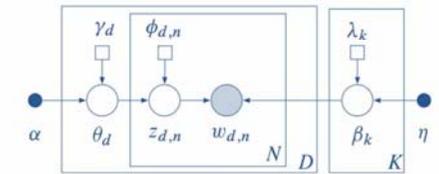


Hoffman, M. D., Blei, D. M., Wang, C. & Paisley, J. 2013. Stochastic variational inference. The Journal of Machine Learning Research, 14, (1), 1303-1347.

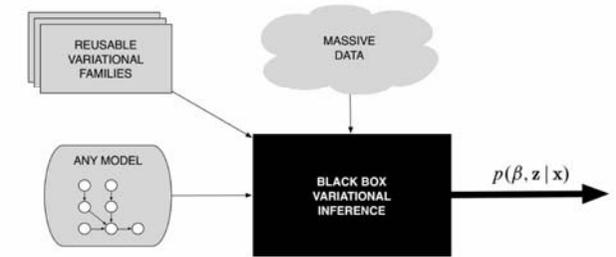


- ▶ Approximate inference can be difficult to derive.
- ▶ Especially true for models that are not conditionally conjugate (Discrete choice models, Bayesian generalized linear models, ...)
- ▶ Holds us back from trying many models.

## 6) Graph Bandits



1. Sample a document
2. Estimate the local variational parameters using the current topics
3. Form intermediate topics from those local parameters
4. Update topics as a weighted average of intermediate and current topics



- ▶ Easily use variational inference with *any model*
- ▶ No exponential family requirements
- ▶ No mathematical work beyond specifying the model

### I'm a bandit

Random topics on optimization, probability, and statistics. By Sébastien Bubeck



Bubeck, S. & Cesa-Bianchi, N. 2012. Regret Analysis of Stochastic and Nonstochastic Multi-armed Bandit Problems. Machine Learning, 5, (1), 1-122.

### ORF523: The complexities of optimization

This page collects together the posts for the graduate course on optimization I taught at Princeton in the Spring 2013. This material has been reorganized (some parts have been cut, some have been added) into a monograph which got recently published: "Foundations and Trends in Machine Learning, Vol. 8, No. 3-4, pp 231-357, 2016" (see here for the free version).

<https://blogs.princeton.edu/imabandit/>

Also very interesting: Bubeck, S. 2015. Convex optimization: Algorithms and complexity. Foundations and Trends in Machine Learning, 8, (3-4), 231-357.



- Slot-machine (bandit - robs your money)
- One-armed bandit
- Very simple model for sequential decision making under uncertainty
- Main challenge: exploration versus exploitation
- Many application domains: A/B-Testing, Crowdsourcing, optimization, search, ...

TU Graz Underlying Principle of the k-Armed Bandits problem

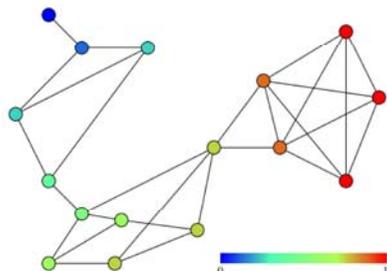
- Let  $a_t \in \{1, \dots, n\}$  be the choice of a machine at time  $t$
- Let  $y_t \in \mathbb{R}$  be the outcome with a mean of  $\langle y_{at} \rangle$
- Now, the given policy maps all history to a new choice:

$$\pi : [(a_1, y_1), (a_2, y_2), \dots, (a_{t-1}, y_{t-1})] \mapsto a_t$$

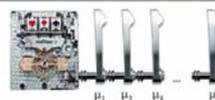
- The problem: Find a policy  $\pi$  that  $\max \langle y_T \rangle$
- Now, two effects appear when choosing such machine:
  - You collect more data about the machine (=knowledge)
  - You collect reward
- Exploration and Exploitation
  - Exploration:** Choose the next action  $a_t$  to  $\min H(b_t)$
  - Exploitation:** Choose the next action  $a_t$  to  $\max \langle y_t \rangle$
- models an agent that simultaneously attempts to acquire new knowledge (called "exploration") and optimize his or her decisions based on existing knowledge (called "exploitation"). The agent attempts to balance these competing tasks in order to maximize total value over the period of time considered.

More information: <http://research.microsoft.com/en-us/projects/bandits>

TU Graz Smooth Graph Function

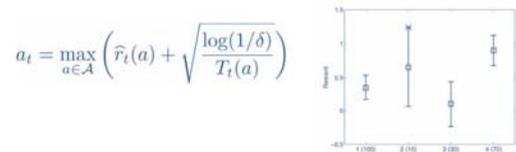


TU Graz Multi-Armed Bandits problem



- Multi-armed bandit:= a gambler strategically operating multiple machines in order to draw the highest possible profits
- There are  $n$  slot-machines ("einarmige Banditen")
- Each machine  $i$  returns a reward  $y \approx P(y; \theta_i)$
- Challenge: The machine parameter  $\theta_i$  is unknown
- Which arm of a slot machine should a gambler pull to maximize his cumulative reward over a sequence of trials? (stochastic setting or adversarial setting)

TU Graz MAP-Principle: "Optimism in the face of uncertainty"



$$a_t = \max_{a \in \mathcal{A}} (\text{rew}_t(a) + \text{uncert}_t(a))$$

**Exploitation**  
the higher the (estimated) reward the higher the chance to select the action

**Exploration**  
the higher the (theoretical) uncertainty the higher the chance to select the action

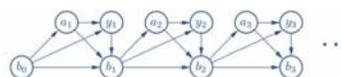
Auer, P., Cesa-Bianchi, N. & Fischer, P. 2002. Finite-time analysis of the multiarmed bandit problem. Machine learning, 47, (2-3), 235-256.

TU Graz Knowledge Representation in MAB

- Knowledge can be represented in two ways:
  - 1) as full history  $h_t = [(a_1, y_1), (a_2, y_2), \dots, (a_{t-1}, y_{t-1})]$
  - or
  - 2) as belief  $b_t(\theta) = P(\theta|h_t)$

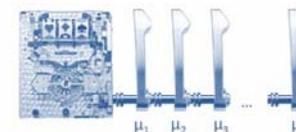
where  $\theta$  are the unknown parameters of all machines

The process can be modelled as belief MDP:



$$P(b'|y, a, b) = \begin{cases} 1 & \text{if } b' = b_{[b, a, y]} \\ 0 & \text{otherwise} \end{cases}, \quad P(y|a, b) = \int_{\theta_a} b(\theta_a) P(y|\theta_a)$$

TU Graz Machine Parameters of the k-armed Bandit



- Each arm  $a$  either wins (reward=1) with fixed (unknown) probability  $\mu_a$ , or loses (reward=0) with fixed (unknown) probability  $1 - \mu_a$
- All draws are independent given  $\mu_1 \dots \mu_k$
  - Problem: How to pull arms to maximize the total reward?

TU Graz A bandit in a graph is still a bandit

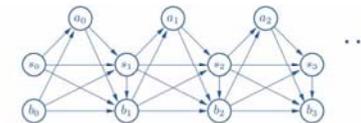
- Let  $G$  a known graph with  $K$  nodes  $\{1, 2, \dots, K\}$
- Let  $f$  be a unknown function defined on the set of nodes
- For  $t = 1$  to  $n$ ,
  - Select a node  $I_t$
  - Observe reward  $r_t = f(I_t) + \epsilon_t$
- Goal: maximize sum of expected rewards
- Equivalently minimize regret:

$$R_n = \sum_{t=1}^n (f^* - f(I_t)),$$

where  $f^* = \max_{1 \leq i \leq K} f(i)$ .

- We care about the case when  $K > n$

TU Graz The optimal policies can be modelled as belief MDP



$$P(b'|s', s, a, b) = \begin{cases} 1 & \text{if } b' = b[s', s, a] \\ 0 & \text{otherwise} \end{cases}, \quad P(s'|s, a, b) = \int_{\theta} b(\theta) P(s'|s, a, \theta)$$

$$V(b, s) = \max_a \left[ E(r|s, a, b) + \sum_{s'} P(s'|s, a, b) V(s', b') \right]$$

Poupart, P., Vlassis, N., Hoey, J. & Regan, K. An analytic solution to discrete Bayesian reinforcement learning. Proceedings of the 23rd international conference on Machine learning, 2006. ACM, 697-704.

- Clinical trials: potential treatments for a disease to select from new patients or patient category at each round, see:

W. Thompson. On the likelihood that one unknown probability exceeds another in view of the evidence of two samples. Bulletin of the American Mathematics Society, vol. 25, pp. 285–294, 1933.

- Games: Different moves at each round, e.g. GO
- Adaptive routing: finding alternative paths, also finding alternative roads for driving from A to B
- Advertisement placements: selection of an ad to display at the Webpage out of a finite set which can vary over time, for each new Web page visitor

- The goal of Standard Randomized Controlled Trials (RCT) are a controlled learning setting:
  - Control for Type I and Type II errors, dependent of trial size  $n_{RCT}$
  - In the case if the patient population  $N$  is smaller than the trial size  $n_{RCT}$ : underpowered trial – problem!
- If we change the goal to
- “learning sufficient - to treat  $N$  as effectively as possible”,
- then **bandit strategies** – optimal policy for max. the expected reward - are perfectly suited!

Kuleshov, V. & Precup, D. 2014. Algorithms for multi-armed bandit problems. *arXiv:1402.6028*.

- 7,000 + different types - more being discovered every day
- >10% of the world population is suffering (if all of the people with rare diseases lived in one country, it would be the world’s 3rd most populous country)
- 80% of rare diseases are genetic, so are present throughout a person’s lifetime, even if symptoms do not immediately appear
- >50% of the people affected by rare diseases are children
- Are responsible for 35% of deaths in the first year of life
- The prevalence distribution is skewed – 80% of all rare disease patients are affected by 350 rare diseases
- >50% of rare diseases do not have a disease specific foundation supporting or researching their rare disease

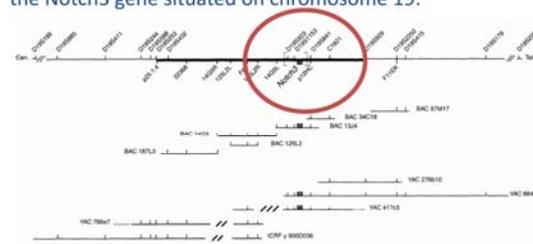
<https://globalgenes.org/rare-diseases-facts-statistics/>  
<https://www.hon.ch/HONselect/RareDiseases/>



<http://fortune.com/2015/10/26/cancer-clinical-trial-belmont-report/>

- Learning → experimenting with all treatments
- Earning → selecting one treatment only, based on experimentation results
- **Question 1: How much learning is best – for an optimal treatment of  $N$  patients?**
- Suppose  $N$  patients with a rare disease:
  - Experimental Group E and control group C
  - e.g. control = response rate  $p_c$  and little information about experimental group
- **Question 2: How many allocations of treatment to E are necessary (= how much experimentation?)**

- Cerebral Autosomal Dominant Arteriopathy with Subcortical Infarcts and Leukoencephalopathy
- is a hereditary disease affecting all the small cerebral arteries. It causes subcortical infarcts and damages the white matter (leukoencephalopathy) and it is due to various mutations of the Notch3 gene situated on chromosome 19:



Joutel, A. et al. 1996. Notch3 mutations in CADASIL, a hereditary adult-onset condition causing stroke and dementia. *Nature*, 383, (6602), 707-710, doi:10.1038/383707a0.

Limitations of drug design for rare diseases due to:

- Lack of understanding of the underlying principles of the rare disease
  - Motivation: Research advances
- Unbalanced economic motivation (cost/benefit)
  - Motivation: Orphan Drug Act and other regulations
- Unavailability of # patients for standard trials
  - This is the true bottleneck!

Villar, S. S., Bowden, J. & Wason, J. 2015. Multi-armed Bandit Models for the Optimal Design of Clinical Trials: Benefits and Challenges. 199-215, doi:10.1214/14-ST5504.

DYNAMIC PROGRAMMING AND LAGRANGE MULTIPLIERS

BY RICHARD BELLMAN

RAND CORPORATION, SANTA MONICA, CALIFORNIA  
 Communicated by Einar Hille, August 15, 1958



Richard Ernest BELLMAN (1920-1984)

1. *Introduction.*—The purpose of this note is to indicate how a suitable combination of the classical method of the Lagrange multiplier and the functional-equation method of the theory of dynamic programming<sup>1</sup> can be used to solve numerically, and treat analytically, a variety of variational problems that cannot readily be treated by either method alone.

A series of applications of the method presented here will appear in further publications.

2. *Functional Equation Approach.*—Consider the problem of maximizing the function

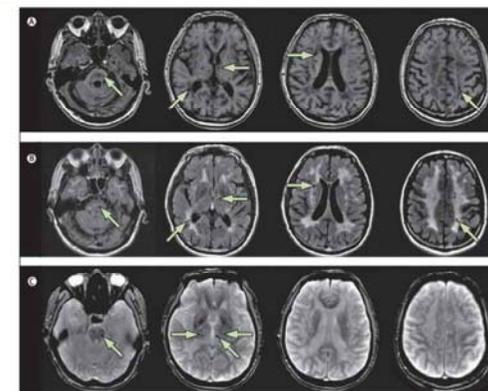
$$F(x_1, x_2, \dots, x_N) = \sum_{i=1}^N g_i(x_i), \quad (2.1)$$

subject to the constraints

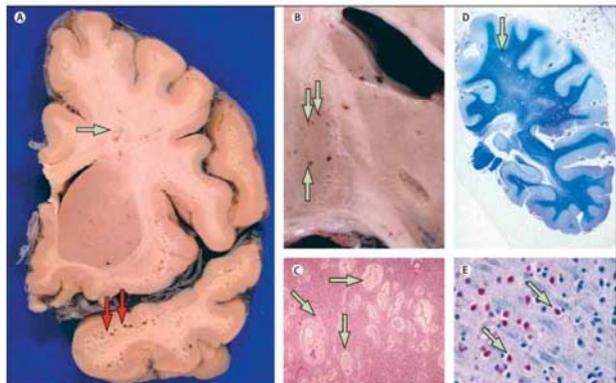
$$(a) \sum_{i=1}^N a_{ij}(x_i) \leq c_j, \quad i = 1, 2, \dots, M, \quad (2.2)$$

$$(b) x_i \geq 0,$$

Bellman, R. 1956. Dynamic programming and Lagrange multipliers. *Proceedings of the National Academy of Sciences*, 42, (10), 767-769.



Chabriat, H., Joutel, A., Dichgans, M., Tournier-Lasserre, E. & Boussier, M.-G. 2009. CADASIL. *The Lancet Neurology*, 8, (7), 643-653, doi:http://dx.doi.org/10.1016/S1474-4422(09)70127-9.



Chabriat, H., Joutel, A., Dichgans, M., Tournier-Lasserre, E. & Bousser, M.-G. 2009. CADASIL. The Lancet Neurology, 8, (7), 643-653, doi:http://dx.doi.org/10.1016/S1474-4422(09)70127-9.



# Thank you!

## Sample Questions (2/3)

- Why do we want to apply ML to graphs?
- Describe typical ML tasks on the example of blood cancer cells!
- If you have a set of points – which similarity measures are useful?
- What is the advantage of factor graphs?
- Why is the Gromov-Hausdorff distance useful?
- What is the central goal of a generative probabilistic model?
- Describe the LDA-model and its application for topic modelling!

# Conclusion and Future Challenges

# Questions

## Sample Questions (2/3)

- Briefly describe the stochastic variational inference algorithms!
- What is the principle of a bandit?
- How does a multi-armed bandit (MAB) work?
- In which ways can a MAB represent knowledge?
- What is the main problem of a clinical trial – and maybe the main problem in clinical medicine?
- Why are rare diseases both important and relevant? Describe an example disease!
- What is the big problem in clinical trials for rare diseases?
- What did Richard Bellman (1956) describe with dynamic programming?
- Why are graph bandits a hot topic for ML research?

- Bandit strategy: Is experimentation worth it for a small number  $N$ ?
- Reconcile clinical trials and clinical practice
- Extensions should deal with randomization, delayed responses and uncertainty around  $N$
- Bayesian bandits need Online-ML
- Bandits are a great source of inspirations and building blocks for solving many problems
- Future work: convex optimization, contextual, combinatorial, ...

Berry, D. A. & Fristedt, B. 1985. Bandit problems: sequential allocation of experiments (Monographs on statistics and applied probability), Springer.

## Sample Questions (1/3)

- What kind of graphical models are used in medical informatics?
- Which type of graph is particularly useful for inference and learning?
- What is the key challenge in the application of graphical models for health informatics?
- What was Judea Pearl (1988) discussing in his paper, for which he received the Turing award?
- What main difficulties arise during breast cancer prognosis?
- What can be done to increase the robustness of prognostic cancer tests?
- Inference in Bayes Nets is NP-complete, but there are certain cases where it is tractable, which ones?

## Solutions of the Quiz

- 1=this is a factor graph of an undirected graph – we have seen this in protein networks (refer to slide Nr. 70 in lecture 5). Factor graph is bipartite and has two types of nodes: Variables, which can be either evidence variables (when we know its value) or query variables (when the value is unknown and we want to predict the value); and factors, which define the relationship between variables in the graph. Each factor can be connected to many variables and comes with a factor function to define the relationship between these variables. For example, if a factor node is connected to two variables nodes A and B, a possible factor function could be  $\text{imply}(A,B)$ , meaning that if the random variable A takes value 1, then so must the random variable B. Each factor function has a weight associated with it, which describes how much influence the factor has on its variables in relative terms. For more information please consult: <http://deepdive.stanford.edu/inference>
- 2= this is the decomposition of a tree, rooted at nodes into subtrees
- 3= metabolic and physical processes that determine the physiological and biochemical properties of a cell. As such, these networks comprise the chemical reactions of metabolism, the metabolic pathways, as well as the regulatory interactions that guide these reactions. With the sequencing of complete genomes, it is now possible to reconstruct the network of biochemical reactions in many organisms, from bacteria to human. Several of these networks are available online: Kyoto Encyclopedia of Genes and Genomes (KEGG)[1], EcoCyc [2], BioCyc [3] and metaTIGER [4]. Metabolic networks are powerful tools for studying and modelling metabolism.
- 4= MYCIN –expert system that used early AI (rule-based) to identify bacteria causing severe infections, such as bacteraemia and meningitis, and to recommend antibiotics, with the dosage adjusted for patient's body weight – the name derived from the antibiotics themselves, as many antibiotics have the suffix “-mycin”.
- 5= Protein-Protein Interaction network (undirected graph here)
- 6= PPI with critical node, bottleneck, hub, etc.

# Appendix

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709.049 06

## Slide 4-19: Biomedical databases ...

HCI-KDD

- ... are libraries of life science data, collected from scientific experiments and computational analyses.
- ... contain (clinical, biological, ...) data from clinical work, genomics, proteomics, metabolomics, microarray gene expression, phylogenetics, etc.
- Examples:
  - Text: e.g. PubMed, OMIM (Online Mendelian Inheritance in Man);
  - Sequence data: e.g. Entrez, GenBank (DNA), UniProt (protein).
  - Protein structures: e.g. PDB, Structural Classification of Proteins (SCOP), CATH (Protein Structure Classification);

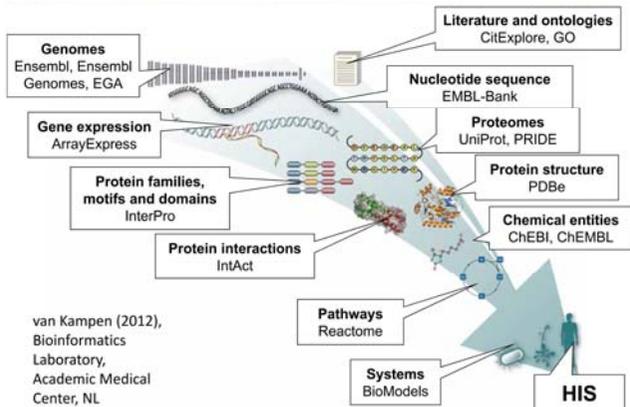
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## Slide 4-21 Databases: From Molecules to Systems

HCI-KDD



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## GM are amongst the most important ML developments

HCI-KDD

- Key Idea: Conditional independence assumptions are very useful – however: Naïve Bayes is extreme!
- X is *conditionally independent* of Y, given Z, if the  $P(X)$  governing X is independent of value Y, given value of Z:

$$(\forall i, j, k) P(X = x_i | Y = y_j, Z = z_k) = P(X = x_i | Z = z_k)$$

can be abbr. with  $P(X|Y, Z) = P(X|Z)$

- Graphical models express sets of conditional independence assumptions via graph structure
- The graph structure plus associated parameters define joint probability distribution over the set of variables

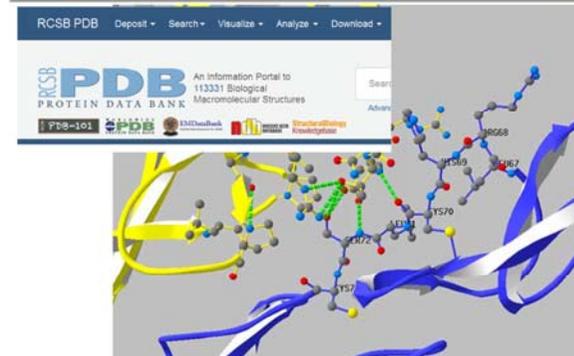
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## Slide 4-20 Example Database: PDB

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Wiltgen, M. & Holzinger, A. (2005) Visualization in Bioinformatics: Protein Structures with Physicochemical and Biological Annotations. In: *Central European Multimedia and Virtual Reality Conference. Prague, Czech Technical University (CTU)*, 69-74

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## Slide 4-22: Example Genome Database: Ensembl

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Ensembl is a joint project between EMBL, EBI and the Wellcome Trust Sanger Institute to develop a software system which produces and maintains automatic generation of annotated genome data across multiple species.

<http://www.ensembl.org/index.html>

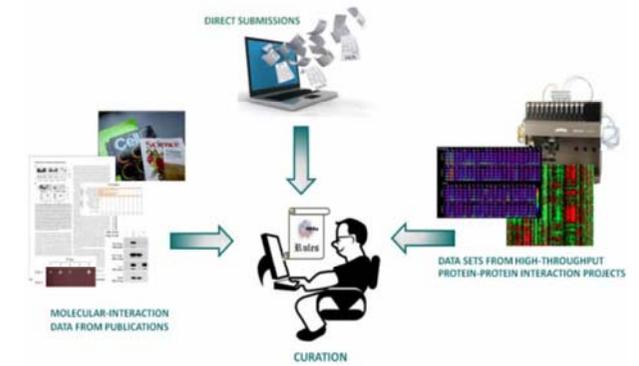
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## Where do the data come from?

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<http://www.ebi.ac.uk/intact/>

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## Slide 4-23 Ex. Gene Expression Database: ArrayExpress

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### Latest News

1 November 2013. **Need to keep your unpublished ArrayExpress microarray data private for longer?** Microarray experiment submitters, have you ever wondered if you could just change the release date of unpublished ArrayExpress data by yourself without emailing curators? Now you can! Use our new release date changing tool (more details on this help page). Submitters of high-throughput sequencing experiments, please continue to email us at [miamexpress@ebi.ac.uk](mailto:miamexpress@ebi.ac.uk) for release date changes so we can make sure the sequence read records at the European Nucleotide Archive are kept in sync.

<http://www.ebi.ac.uk/arrayexpress/>

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