


Andreas Holzinger
185.A83 Machine Learning for Health Informatics
2019S, VU, 2.0 h, 3.0 ECTS
Dienstag, 30. April 2019



From Data for Machine Learning to probabilistic information and entropy

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<https://human-centered.ai/machine-learning-for-health-informatics-class-2019>



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1

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- 01 Data – the underlying physics of data
- 02 Biomedical data sources – taxonomy of data
- 03 Data integration, mapping, fusion
- 04 Probabilistic Information
- 05 Information Theory – Information Entropy
- 06 Cross- Entropy - Kullback-Leibler Divergence



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2

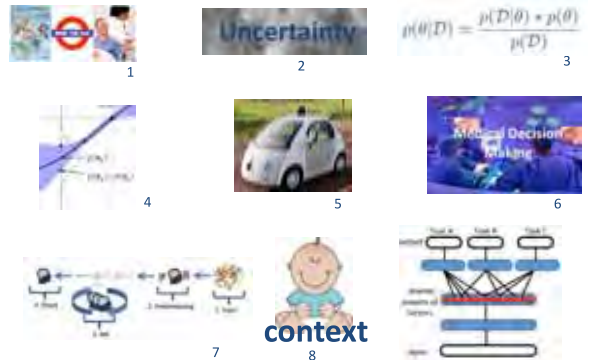
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Image source: <http://www.hutu6.com/reflection-wallpapers.html>

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Image source: <http://www.efmc.info/medchemwatch-2014-1/lab.php>

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Domingos, P. 2015. The Master Algorithm: How the Quest for the Ultimate Learning Machine Will Remake Our World, Penguin UK.

- What does “big data” mean? Is it good or bad?
- Would you collect more data of low quality?
- Or would you use only data of high quality?
- What is data quality?
- How do you measure data quality?
- What about data protection and privacy?
- What about data security?
- What does data accessibility mean?
- What does interpretability need?

Leo L. Pipino, Yang W. Lee & Richard Y. Wang 2002. Data quality assessment. Communications of the ACM, 45, (4), 211-218.

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6

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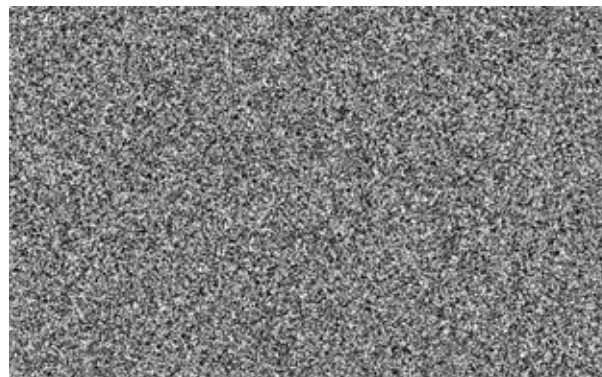
- Heterogeneous, distributed, inconsistent data sources (need for **data integration** & fusion) [1]
- **Complex data** (high-dimensionality – challenge of dimensionality reduction and visualization) [2]
- Noisy, uncertain, missing, dirty, and imprecise, imbalanced data (challenge of **pre-processing**)
- The discrepancy between data-information-knowledge (**various definitions**)
- **Big data** sets in high-dimensions (manual handling of the data is often impossible) [3]

1. Holzinger A, Dehmer M, & Jurisica J (2014) Knowledge Discovery and Interactive Data Mining in Bioinformatics - State-of-the-Art, future challenges and research directions. BMC Bioinformatics 15(S6):11.
2. Hund, M., Sturm, W., Schreck, T., Ullrich, T., Keim, D., Majnarić, L. & Holzinger, A. 2015. Analysis of Patient Groups and Immunization Results Based on Subspace Clustering. In: LNAI 9250, 358-368.
3. Holzinger, A., Stocker, C. & Dehmer, M. 2014. Big Complex Biomedical Data: Towards a Taxonomy of Data. in CCIS 455. Springer 3-18.

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8

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01 The underlying physics of data

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9

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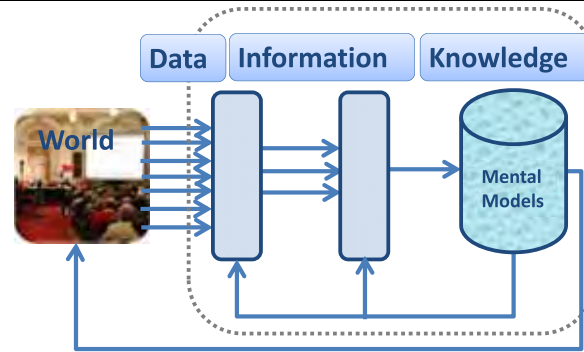
- Data in traditional Statistics
- Low-dimensional data ($< \mathbb{R}^{100}$)
- Problem: Much noise in the data
- Not much structure in the data but it can be represented by a simple model
- Data in Machine Learning
- High-dimensional data ($>> \mathbb{R}^{100}$)
- Problem: not noise, but complexity
- Much structure, but the structure can **not** be represented by a simple model

Lecun, Y., Bengio, Y. & Hinton, G. 2015. Deep learning. Nature, 521, (7553), 436-444.

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10

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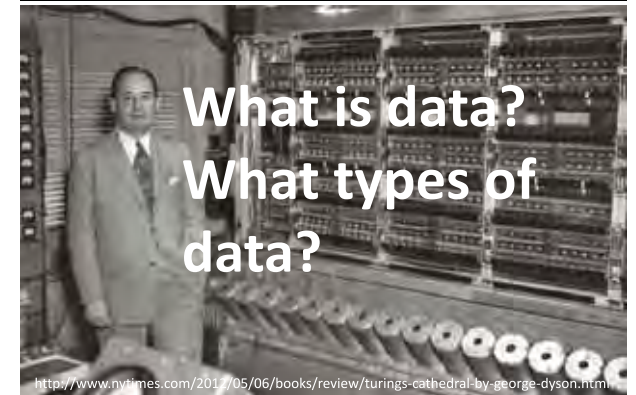


Knowledge := a set of expectations

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11

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<http://www.nytimes.com/2017/05/06/books/review/turings-cathedral-by-george-dyson.html>

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12

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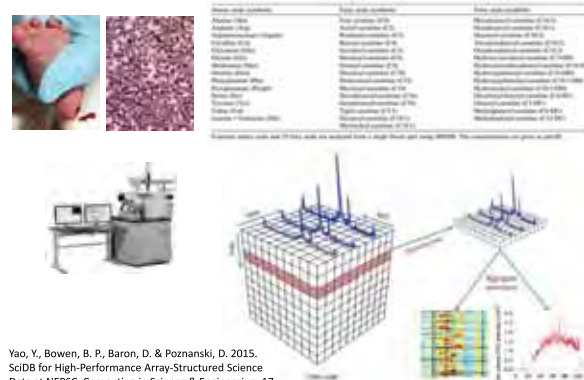


http://www.nlm.nih.gov/cgi/mesh/2011/MB_cgi?mode=&index=15177&view=expanded#TreeE01.370.500.580

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13

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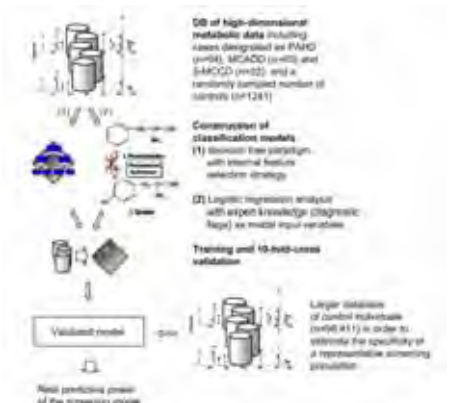


Yao, Y., Bowen, B. P., Baron, D. & Poznanski, D. 2015. SciDB for High-Performance Array-Structured Science Data at NERSC. Computing in Science & Engineering, 17, (3), 44-52, doi:10.1109/MCSE.2015.43.

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14

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Baumgartner, C., Bohm, C. & Baumgartner, D. 2005. Modelling of classification rules on metabolic patterns including machine learning and expert knowledge. Journal of Biomedical Informatics, 38, (2), 89-98, doi:10.1016/j.jbi.2004.08.009.

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15

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02 Biomedical data sources: Taxonomy of data



10-12

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17

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Karp, G. 2010. Cell and Molecular Biology: Concepts and Experiments, Gainesville, John Wiley.

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18

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16

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bionumbers.hms.harvard.edu/

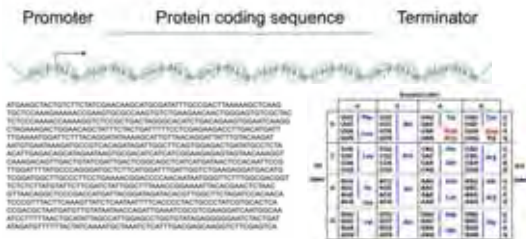
<http://book.bionumbers.org/how-many-genes-are-in-a-genome/>

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Organism	# of protein-coding genes	# of genes (native estimate)	Genome size (Mb)
HIV 1	9	10	115,000
Influenza A virus	10-11	14	120,000
Bacteriophage ϕ	46	46	120,000
Egyptian Bat virus	80	120	120,000
Buchnera sp.	6,100	640	120,000
<i>C. parvulus</i>	1,900	1,900	120,000
<i>S. aureus</i>	2,700	2,900	120,000
<i>M. luteus</i>	3,900	4,000	120,000
<i>E. coli</i>	4,300	4,200	11,440
<i>E. coli</i>	4,300	4,600	10,940
<i>S. cerevisiae</i>	6,600	12,000	10,444
<i>C. elegans</i>	20,000	100,000	101,964
<i>A. thaliana</i>	27,000	140,000	11,790
<i>D. melanogaster</i>	14,000	140,000	11,130
<i>F. rubripes</i>	19,000	400,000	11,125
<i>Z. mays</i>	33,000	2,300,000	11,000
<i>M. musculus</i>	31,000	2,300,000	10,000
<i>H. sapiens</i>	21,000	2,200,000	11,150
<i>T. aestivum</i> (wheat)	95,000	16,800,000	10,400

19

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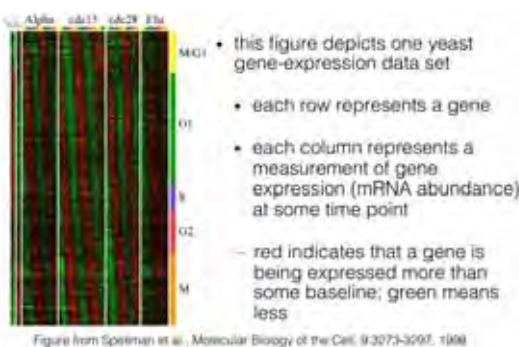
For further reading this is recommended:
Buffalo, V. 2015. Bioinformatics Data Skills: Reproducible and Robust Research with Open Source Tools, Sebastopol (CA), O'Reilly.



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22

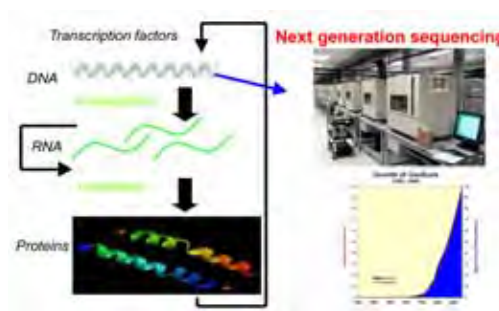
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25

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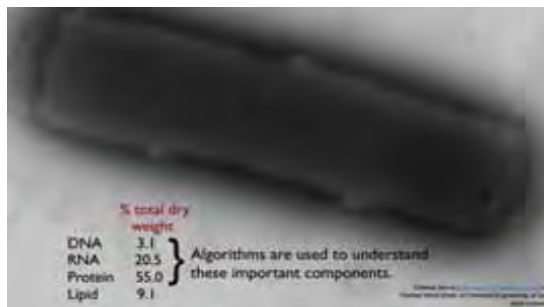


Navlakha, S. & Bar-Joseph, Z. 2011. Algorithms in nature: the convergence of systems biology and computational thinking. *Molecular Systems Biology*, 7.

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20

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23

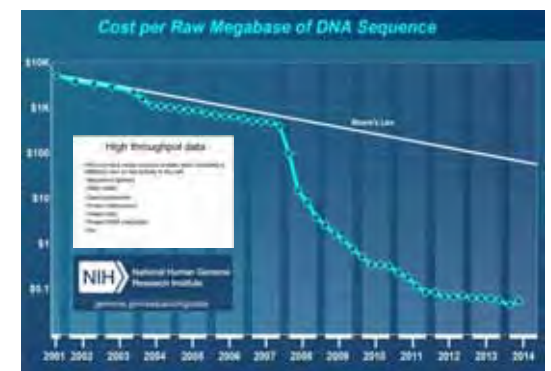
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- Physical level -> bit = binary digit = basic indissoluble unit (= Shannon, Sh), ≠ Bit (!) in Quantum Systems -> qubit
- Logical Level -> integers, booleans, characters, floating-point numbers, alphanumeric strings, ...
- Conceptual (Abstract) Level -> data-structures, e.g. lists, arrays, trees, graphs, ...
- Technical Level -> Application data, e.g. text, graphics, images, audio, video, multimedia, ...
- "Hospital Level" -> Narrative (textual) data, numerical measurements (physiological data, lab results, vital signs, ...), recorded signals (ECG, EEG, ...), Images (x-ray, MR, CT, PET, ...) ; -omics

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- Billions of biological data sets are openly available, here only some examples:
- General Repositories:
 - GenBank, EMBL, HMCA, ...
- Specialized by data types:
 - UniProt/SwissProt, MMMP, KEGG, PDB, ...
- Specialized by organism:
 - WormBase, FlyBase, NeuroMorpho, ...
- Details: <http://hci-kdd.org/open-data-sets>



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24

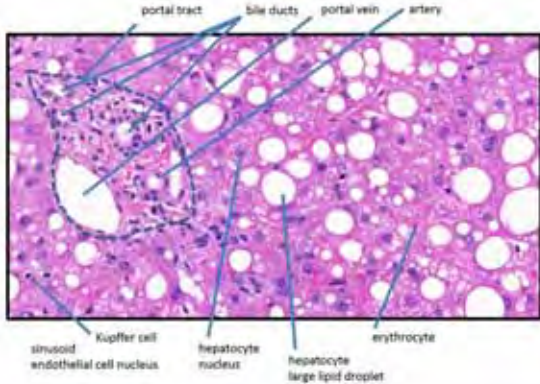
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- Clinical workplace data sources
 - Medical documents: text (non-standardized ("free-text"), semi-structured, standard terminologies (ICD, SNOMED-CT)
 - Measurements: lab, time series, ECG, EEG, EOG, ...
 - Surveys, Clinical study data, trial data
- Image data sources
 - Radiology: MRI (256x256, 200 slices, 16 bit per pixel, uncompressed, ~26 MB); CT (512x512, 60 slices, 16 bit per pixel, uncompressed ~32MB; MR, US;
 - Digital Microscopy : WSI (15mm slide, 20x magn., 24 bits per pixel, uncompressed, 2,5 GB, WSI 10 GB; confocal laser scanning, etc.
- omics data sources
 - Sanger sequencing, NGS whole genome sequencing (3 billion reads, read length of 36) ~ 200 GB; NGS exome sequencing ("only" 110,000,000 reads, read length of 75) ~7GB; Microarray, mass-spectrometry, gas chromatography, ...

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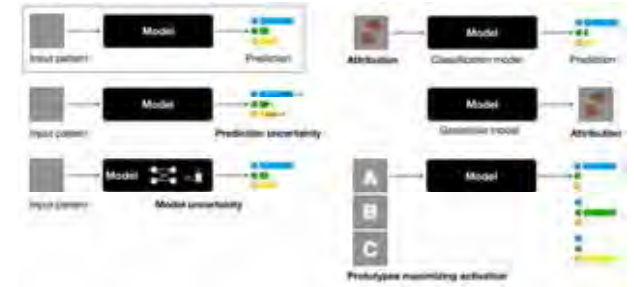
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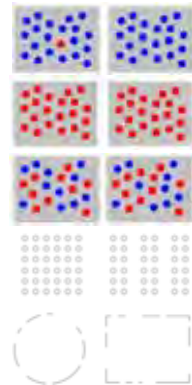


- Level 1. Association $P(y|x)$ with the typical activity of "seeing" and questions including "How would seeing X change my belief in Y?", in our use-case above this was the question of "what does a feature in a histology slide the pathologist about a disease?"
- Level 2. Intervention $P(y|do(x))$ with the typical activity of "doing" and questions including "What if I do X?", in our use-case above this was the question of "what if the medical professional recommends treatment X - will the patient be cured?"
- Level 3. Counterfactuals $P(y_x|x', y')$ with the typical activity of "retrospection" and questions including "Was Y the cause for X?", in our use-case above this was the question of "was it the treatment that cured the patient?"

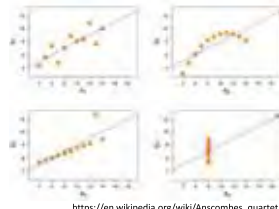
Andreas Holzinger, Georg Langs, Helmut Denk, Kurt Zatloukal & Heimo Mueller 2019. Causability and Explainability of AI in Medicine. Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery, doi:10.1002/widm.1312.



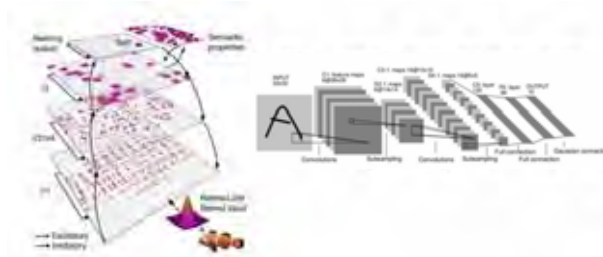
Andreas Holzinger, Georg Langs, Helmut Denk, Kurt Zatloukal & Heimo Mueller 2019. Causability and Explainability of AI in Medicine. Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery, doi:10.1002/widm.1312.



Kurt Koffka 1935. Principles of Gestalt Psychology, New York, Harcourt.

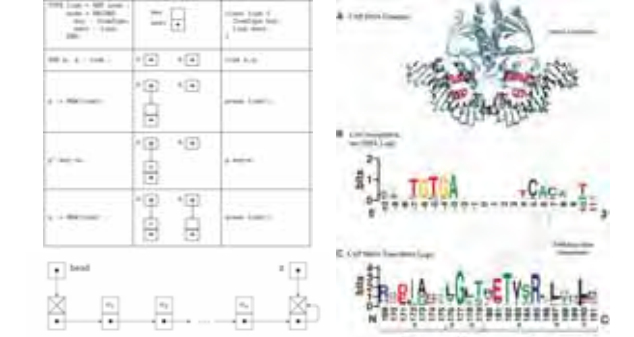


https://en.wikipedia.org/wiki/Anscombes_quartet



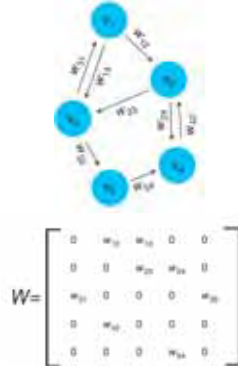
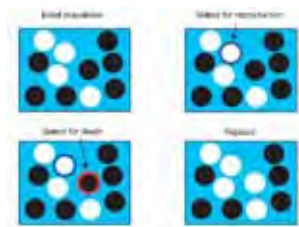
David H. Hubel & Torsten N. Wiesel 1962. Receptive fields, binocular interaction and functional architecture in the cat's visual cortex. The Journal of Physiology, 160, (1), 106-154, doi:10.1113/jphysiol.1962.sp006837.

Yann Lecun, Bernhard Boser, John S. Denker, Donnie Henderson, Richard E. Howard, Wayne Hubbard & Lawrence D. Jackel 1989. Backpropagation applied to handwritten zip code recognition. Neural computation, 1, (4), 541-551, doi:10.1162/neco.1989.1.4.541.



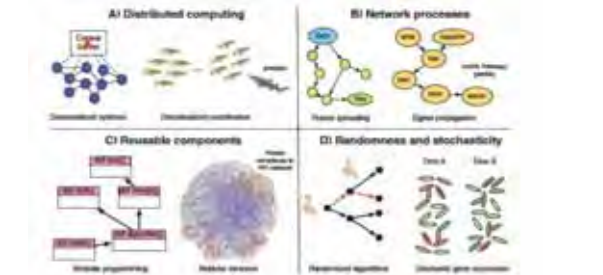
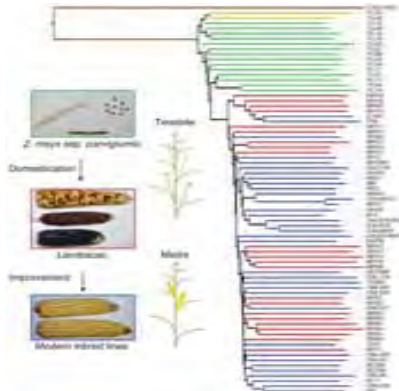
Crooks, G. E., Hon, G., Chandonia, J. M. & Brenner, S. E. (2004) WebLogo: A sequence logo generator. Genome Research, 14, 6, 1188-1190.

Evolutionary dynamics act on populations. Neither genes, nor cells, nor individuals evolve; only populations evolve.



Lieberman, E., Hauert, C. & Nowak, M. A. (2005) Evolutionary dynamics on graphs. Nature, 433, 7023, 312-316.

Hufford et. al. 2012. Comparative population genomics of maize domestication and improvement. Nature Genetics, 44, (7), 808-811.



<http://cacm.acm.org/magazines/2015/1/181614-distributed-information-processing-in-biological-and-computational-systems/abstract>

Navlakha, S. & Bar-Joseph, Z. 2014. Distributed information processing in biological and computational systems. Commun. ACM, 58, (1), 94-102.

<https://www.youtube.com/watch?v=4u47nwHq4&feature=youtu.be>

- Grand Challenges in this area:
 - Production of Open Data Sets
 - Synthetic data sets for learning algorithm testing
 - Privacy preserving machine learning
 - Data leak detection
 - Data citation
 - Differential privacy
 - Anonymization and pseudonymization
 - Evaluation and benchmarking

Please visit:
<http://hci-kdd.org/privacy-aware-machine-learning-for-data-science/>

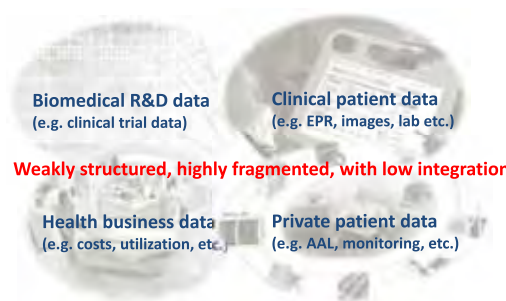


03 Data Integration, mapping, fusion



Our central hypothesis: Information may bridge this gap

Holzinger, A. & Simon, K.-M. (eds.) 2011. *Information Quality in e-Health. Lecture Notes in Computer Science LNCS 7058*, Heidelberg, Berlin, New York: Springer.

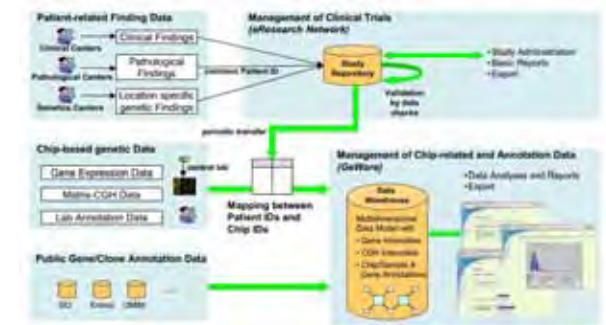


Manyika, J., Chui, M., Brown, B., Bughin, J., Dobbs, R., Roxburgh, C. & Byers, A. H. (2011) *Big data: The next frontier for innovation, competition, and productivity*. Washington (DC), McKinsey Global Institute.

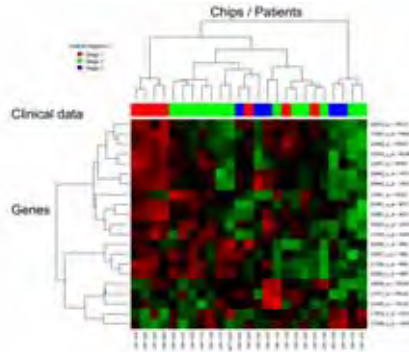
Unsolved Problem: Data Integration and Data Fusion in the Life Sciences

How to combine these different data types together to obtain a unified view of the activity in the cell is one of the major challenges of systems biology

Navlakha, S. & Bar-Joseph, Z. 2014. Distributed information processing in biological and computational systems. *Commun. ACM*, 58, (1), 94-102, doi:10.1145/2678280.



Kirsten, T., Lange, J. & Rahm, E. 2006. An integrated platform for analyzing molecular-biological data within clinical studies. *Current Trends in Database Technology-EDBT 2006*. Heidelberg: Springer, pp. 399-410, doi:10.1007/11896548_31.



Kirsten, T., Lange, J. & Rahm, E. 2006. An integrated platform for analyzing molecular-biological data within clinical studies. *Current Trends in Database Technology-EDBT 2006*. Heidelberg: Springer, pp. 399-410, doi:10.1007/11896548_31.

- Genomics (sequence annotation)
- Transcriptomics (microarray)
- Proteomics (Proteome Databases)
- Metabolomics (enzyme annotation)
- Fluxomics (isotopic tracing, metabolic pathways)
- Phenomics (biomarkers)
- Epigenomics (epigenetic modifications)
- Microbiomics (microorganisms)
- Lipidomics (pathways of cellular lipids)

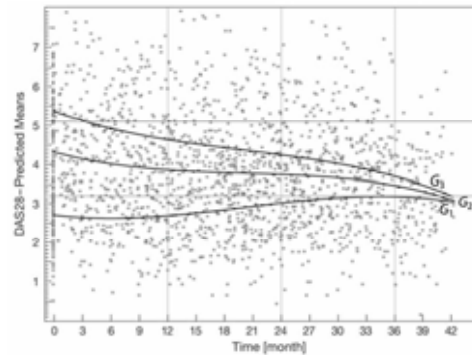


Joyce, A. R. & Palsson, B. Ø. 2006. The model organism as a system: integrating 'omics' data sets. *Nature Reviews Molecular Cell Biology*, 7, 198-210.

- 50+ Patients per day ~ 5000 data points per day ...
- Aggregated with specific scores (Disease Activity Score, DAS)
- Current patient status is related to previous data
- = convolution over time
- ⇒ **time-series data**



Simonik, K. M., Holzinger, A., Bloice, M. & Hermann, J. (2011). *Optimizing Long-Term Treatment of Rheumatoid Arthritis with Systematic Documentation. Pervasive Health - 5th International Conference on Pervasive Computing Technologies for Healthcare, Dublin, IEEE, 550-554.*



Simonik, K. M., Holzinger, A., Bloice, M. & Hermann, J. (2011). *Optimizing Long-Term Treatment of Rheumatoid Arthritis with Systematic Documentation. Pervasive Health - 5th International Conference on Pervasive Computing Technologies for Healthcare, Dublin, IEEE, 550-554.*

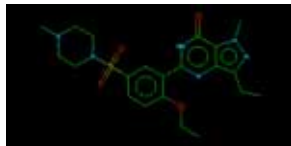
- 0-D data = a data point existing isolated from other data, e.g. integers, letters, Booleans, etc.
- 1-D data = consist of a string of 0-D data, e.g. Sequences representing nucleotide bases and amino acids, SMILES etc.
- 2-D data = having spatial component, such as images, NMR-spectra etc.
- 2.5-D data = can be stored as a 2-D matrix, but can represent biological entities in three or more dimensions, e.g. PDB records
- 3-D data = having 3-D spatial component, e.g. image voxels, e-density maps, etc.
- H-D Data = data having arbitrarily high dimensions

SMILES (Simplified Molecular Input Line Entry Specification)

... is a compact machine and human-readable chemical nomenclature:

e.g. Viagra:

CCc1nn(C)c2c(=O)[nH]c(nc12)c3cc(ccc3OCC)S(=O)(=O)N4CCN(C)CC4

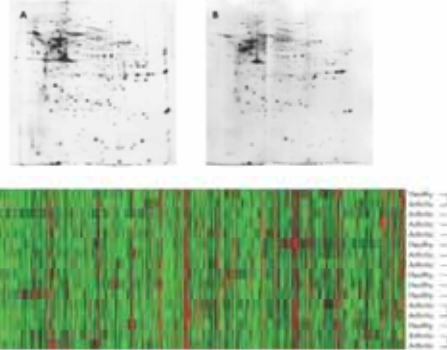


...is Canonicalizable

...is Comprehensive

...is Well Documented

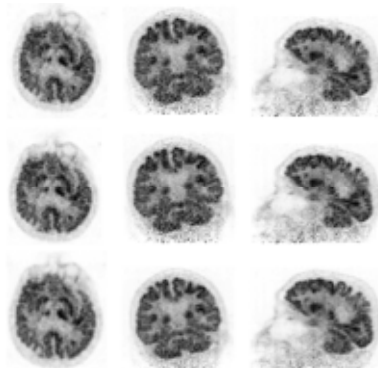
http://www.daylight.com/dayhtml_tutorials/languages/smiles/index.html



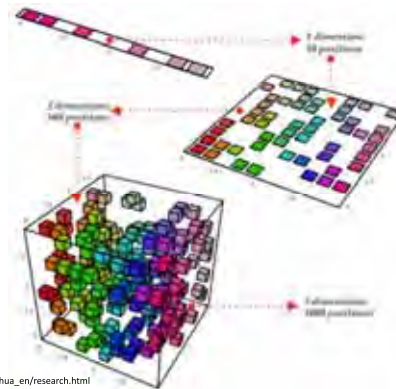
Kastrinaki et al. (2008) Functional, molecular & proteomic characterisation of bone marrow mesenchymal stem cells in rheumatoid arthritis. *Annals of Rheumatic Diseases*, 67, 6, 741-749.



<http://www.pdb.org>



Scheins, J. J., Herzog, H. & Shah, N. J. (2011) Fully-3D PET Image Reconstruction Using Scanner-Independent, Adaptive Projection Data and Highly Rotation-Symmetric Voxel Assemblies. *Medical Imaging, IEEE Transactions on*, 30, 3, 879-892.



Bengio, S. & Bengio, Y. 2000. Taking on the curse of dimensionality in joint distributions using neural networks. *IEEE Transactions on Neural Networks*, 11, (3), 550-557.

http://www.iro.umontreal.ca/~bengio/yoshua_en/research.html

- Bridging the gap between natural sciences and clinical medicine (who has seen genomics and patient data integrated in routine???)
- Organizational barriers, data provenance, data ownership, privacy, accessibility, usability, fair use of data, security, safety, data protection
- Combine Ontologies with Machine Learning
- Stochastic Ontologies, Ontology learning
- Integration of data from wet-labs with in-silico experimental data (e.g. tumor growth simulation)

04 Probabilistic Information p(x)

- **Communication** (Hartley, Nyquist, Shannon)
 - **Coding Theory** (Fano, Hamming, Reed, Solomon)
 - **Cryptography** (Hellman, Rivest, Shamir, Adleman)
 - **Complexity** (Kolmogorov, Chaitin) **Computation, Chaos**
 - **Cybernetics** (Wiener, von Neumann, Langton)
 - **Foundations** (Brillouin, Bennet, Landauer)
 - **Canonical Quantum Gravity** (Wheeler, De-Witt)
 - **Metabiology** (Conrad, Chaitin)
 - **Unification via Information** (Carlo Rovelli's books)
- Universe's ultimate mechanism for existence might be information: "it from bit" (Wheeler's last speculation)

Manca, V. 2013. Infobiotics: Information in Biotic Systems, Heidelberg, Springer, doi:10.1007/978-3-642-36223-1.

The inverse probability allows to infer unknowns, learn from data and make **predictions**:

1) Maximum-Likelihood Learning

finds a parameter setting, that maximizes the p(x) of the data: $P(D|\theta)$

2) Maximum a Posteriori Learning (e.g. for MCMC)

assumes a prior over the model parameters $P(\theta)$ and finds a parameter setting that maximizes the posterior: $P(\theta|D) \propto P(\theta)P(D|\theta)$

3) Bayesian Learning

assumes a prior over the model parameters and computes the posterior distribution $P(\theta|D)$

- Boolean models
- Algebraic models
- Probabilistic models *)

*) Our probabilistic models describes data which we can observe from our environment – and if we use the mathematics of probability theory, in order to express the uncertainties around our model then the inverse probability allows us to infer unknown unknowns ... learning from data and making predictions – the core essence of machine learning and of vital importance for health informatics

Ghahramani, Z. 2015. Probabilistic machine learning and artificial intelligence. *Nature*, 521, (7553), 452-459, doi:10.1038/nature14541.

Probabilistic Information p(x)



Thomas Bayes
1701 - 1761

Bayes, T. (1763). An Essay towards solving a Problem in the Doctrine of Chances (Postum communicated by Richard Price). *Philosophical Transactions*, 53, 370-418.

$$p(x_i) = \sum P(x_i, y_j) \quad p(x_i, y_j) = p(y_j|x_i)P(x_i)$$

Bayes' Rule is a corollary of the Sum Rule and Product Rule:

$$p(x_i|y_j) = \frac{p(y_j|x_i)p(x_i)}{\sum p(x_i, y_j)p(x_i)}$$

Barnard, G. A., & Bayes, T. (1958). Studies in the history of probability and statistics: IX. Thomas Bayes's essay towards solving a problem in the doctrine of chances. *Biometrika*, 45(3/4), 293-315.

General setting:

- Given a (hypothesized & probabilistic) model that governs the random experiment
- The model gives a probability of any data $p(D|\theta)$ that depends on the parameter θ
- Now, given actual sample data $X = \{x_1, \dots, x_n\}$, what can we say about the value of θ ?

Intuitively, take your best guess of θ

▪ "best" means "best explaining/fitting the data"

▪ Generally an optimization problem



Lane, N. & Martin, W. (2010) The energetics of genome complexity. *Nature*, 467, 7318, 929-934.

Bayes' Rule in words

$d \dots$ data; $h \dots$ hypothesis

$H = \{H_1, H_2, \dots, H_n\} \dots$ Hypothesis space

$\forall h, d \dots$

$$p(h|d) = \frac{p(d|h)p(h)}{\sum_{h \in H} p(d|h')p(h')}$$

Posterior Probability

Likelihood

Prior Probability

Sum over space of alternative hypotheses

Evidence = marginal likelihood

1) Maximum likelihood estimation (given X)

- "Best" means "data likelihood reaches maximum"

$$\hat{\theta} = \arg \max_{\theta} P(X|\theta)$$

- **Problem: massive amount of data necessary**

2) Bayesian estimation (use posterior)

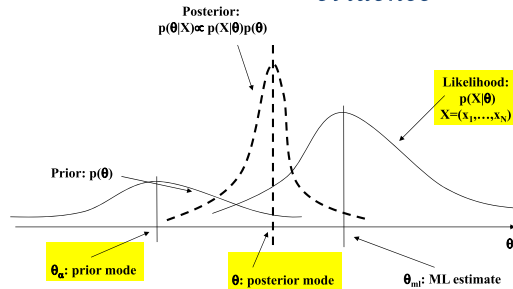
$$\hat{\theta} = \arg \max_{\theta} P(X|\theta) = \arg \max_{\theta} P(X|\theta)P(\theta)$$

- "Best" means being consistent with our "prior" knowledge and explaining data well

- **Problem: how to define prior?**

An example can be found in: Banerjee, O., El Ghaoui, L. & D'aspremont, A. 2008. Model selection through sparse maximum likelihood estimation for multivariate gaussian or binary data. *The Journal of Machine Learning Research*, 9, 485-516. Available via: <http://arxiv.org/pdf/0707.0704>

$$\text{posterior } p(x) = \frac{\text{likelihood} * \text{prior } p(x)}{\text{evidence}}$$



For more basic information: Bishop, C. M. 2007. *Pattern Recognition and Machine Learning*. Springer.
For application examples in Text processing refer to: Jiang, J. & Zhai, C. X. 2007. An empirical study of tokenization strategies for biomedical information retrieval. *Information Retrieval*, 10, (4-5), 341-363.
Holzinger Group, human-centered.ai 64 MAKE Health Lecture 06

05 Information Theory & Entropy

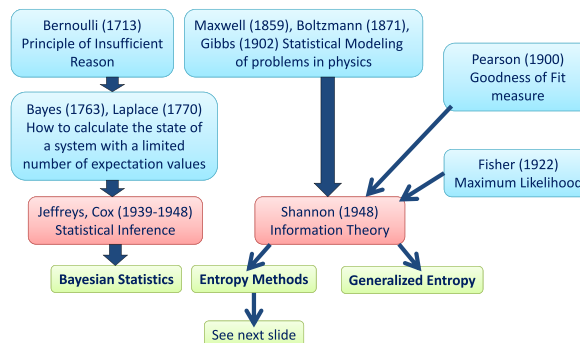
- Information is the reduction of uncertainty
- If something is 100 % certain its uncertainty = 0
- Uncertainty is max. if all choices are equally probable (I.I.D)
- Uncertainty (as information) sums up for independent sources

Entropy as measure for disorder



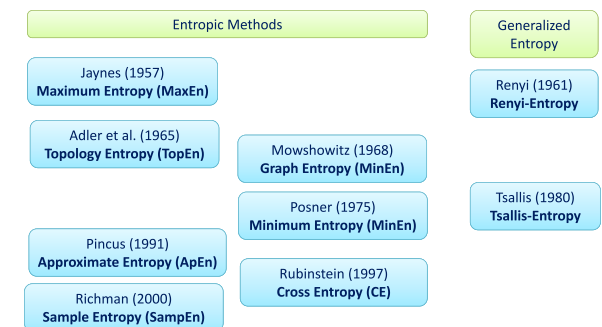
<http://www.scottaaronson.com>

An overview on the History of Entropy



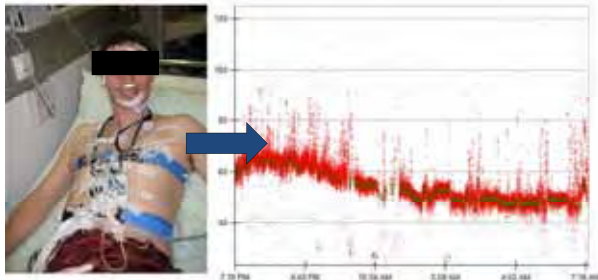
confer also with: Golan, A. (2008) Information and Entropy Econometric: A Review and Synthesis. *Foundations and Trends in Econometrics*, 2, 1-2, 1-145.

Towards a Taxonomy of Entropic Methods



Holzinger, A., Hörtenhuber, M., Mayer, C., Bachler, M., Wassertheurer, S., Pinho, A. & Koslicki, D. 2014. On Entropy-Based Data Mining. In: Holzinger, A. & Jurisica, I. (eds.) *Lecture Notes in Computer Science, LNCS 8401*. Berlin Heidelberg: Springer, pp. 209-226.

Example of the usefulness of ApEn (1/3)



Holzinger, A., Stocker, C., Bruschi, M., Auinger, A., Silva, H., Gamboa, H. & Fred, A. 2012. On Applying Approximate Entropy to ECG Signals for Knowledge Discovery on the Example of Big Sensor Data. In: Huang, R., Ghorbani, A., Pasi, G., Yamaguchi, T., Yen, N. & Jin, B. (eds.) *Active Media Technology, Lecture Notes in Computer Science, LNCS 7669*. Berlin Heidelberg: Springer, pp. 646-657.

EU Project EMERGE (2007-2010)

Example of the usefulness of ApEn (2/3)

$$\text{Let: } \langle x_n \rangle = \{x_1, x_2, \dots, x_N\}$$

$$\vec{x}_i = (x_i, x_{i+1}, \dots, x_{i+m-1})$$

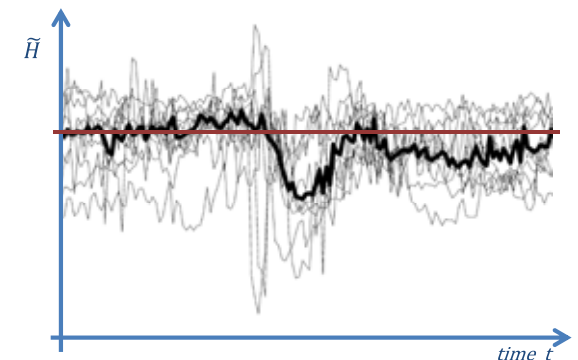
$$\|\vec{x}_i, \vec{x}_j\| = \max_{k=1,2,\dots,m} (|x_{i+k-1} - x_{j+k-1}|)$$

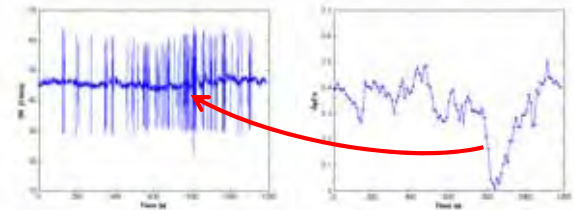
$$\tilde{H}(m, r) = \lim_{N \rightarrow \infty} [\phi^m(r) - \phi^{m+1}(r)]$$

$$C_r^m(i) = \frac{N^m(i)}{N - m + 1} \quad \phi^m(r) = \frac{1}{N - m + 1} \sum_{t=1}^{N-m+1} \ln C_r^m(i)$$

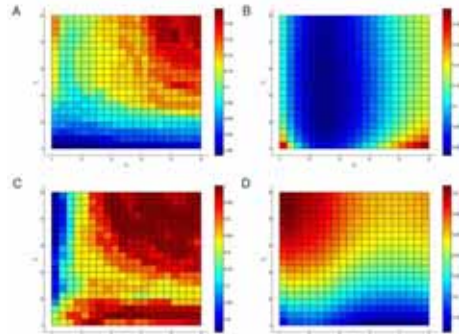
Pincus, S. M. (1991) Approximate Entropy as a measure of system complexity. *Proceedings of the National Academy of Sciences of the United States of America*, 88, 6, 2297-2301.

Example: ApEn (2)





Holzinger, A., Hörtenhuber, M., Mayer, C., Bachler, M., Wassertheurer, S., Pinho, A. & Koslicki, D. 2014. On Entropy-Based Data Mining. In: Holzinger, A. & Jurisica, I. (eds.) Interactive Knowledge Discovery and Data Mining in Biomedical Informatics, Lecture Notes in Computer Science, LNCS 8401. Berlin Heidelberg: Springer, pp. 209-226.

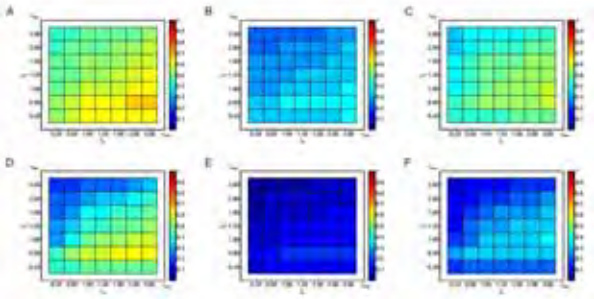


Mayer, C., Bachler, M., Hörtnerhuber, M., Stocker, C., Holzinger, A. & Wassertheurer, S. 2014. Selection of entropy-measure parameters for knowledge discovery in heart rate variability data. *BMC Bioinformatics*, 15, (Suppl 6), S2, doi:doi:10.1186/1471-2105-15-S6-S2.



- Heart Rate Variability (HRV) can be used as a marker of cardiovascular health status.
- Entropy measures represent a family of new methods to quantify the variability of the heart rate.
- Promising approach, due to ability to discover certain patterns and shifts in the "apparent ensemble amount of randomness" of stochastic processes,
- measure randomness and **predictability of processes**.

Mayer, C., Bachler, M., Holzinger, A., Stein, P. K. & Wassertheurer, S. 2016. The Effect of Threshold Values and Weighting Factors on the Association between Entropy Measures and Mortality after Myocardial Infarction in the Cardiac Arrhythmia Suppression Trial (CAST). *Entropy*, 18, (4), 129, doi:10.3390/e18040129.



Mayer, C., Bachler, M., Holzinger, A., Stein, P. K. & Wassertheurer, S. 2016. The Effect of Threshold Values and Weighting Factors on the Association between Entropy Measures and Mortality after Myocardial Infarction in the Cardiac Arrhythmia Suppression Trial (CAST). *Entropy*, 18, (4), 129, doi:10.3390/e18040129.

06 Cross-Entropy Kullback-Leibler Divergence

- Entropy:
 - Measure for the **uncertainty** of random variables
- Kullback-Leibler divergence:
 - comparing two distributions**
- Mutual Information:
 - measuring the **correlation** of two random variables

ON INFORMATION AND SUFFICIENCY
By S. KULLBACK AND R. A. LEIBLER
The George Washington University and Washington, D. C.

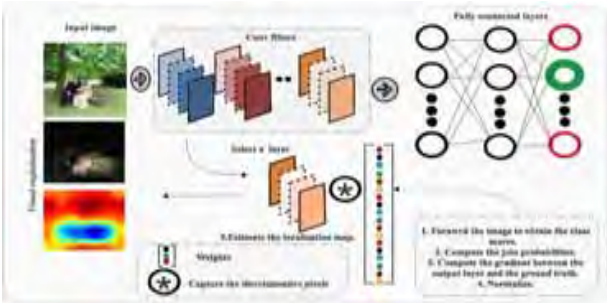
1. Introduction. This note generalizes to the abstract case Shannon's definition of information [10], [16]. Wiener's information (p. 75 of [18]) is essentially the same as Shannon's although their motivation was different (cf. footnote 1, p. 95 of [16]) and Shannon apparently has investigated the concept more completely. R. A. Fisher's definition of information (intrinsic accuracy) is well known (p. 299 of [9]). However, his concept is quite different from that of Shannon and Wiener, and hence ours, although the two are not unrelated as is shown in paragraph 2.

R. A. Fisher, in his original introduction of the criterion of efficiency, required "that the statistic chosen should summarize the whole of the relevant information supplied by the sample," (p. 292 of [9]). Halmos and Savage in a recent paper, one of the main results of which is a generalization of the well known Fisher-Neyman theorem on sufficient statistics to the abstract case, conclude, "We think that confusion has from time to time been thrown on the subject by . . . and (ii) the assumption that a sufficient statistic contains all the information is only the technical sense of 'information' as measured by variance," (p. 241 of [8]). It is shown in this note that the information in a sample as defined herein, that is, in the Shannon-Wiener sense cannot be increased by any statistical operations and is invariant (not decrease) if and only if sufficient statistics are employed. For a similar property of Fisher's information see p. 717 of [9], Doob [19].

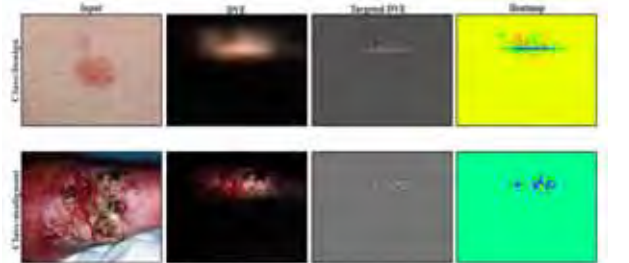
We are also concerned with the statistical problem of discrimination [21], [17], by considering a measure of the "distance" or "divergence" between statistical populations [21], [21], [13] in terms of our measure of information. For the statistician two populations differ more or less according as to how difficult it is to discriminate between them with the best test [16]. The particular measure of divergence we use has been considered by Jeffreys [20], [13] in another connection. He is primarily concerned with its use in providing an invariant density of a prior probability. A special case of this divergence is Mahalanobis' generalized distance [19].

Solomon Kullback 1907-1994 Richard Leibler 1914-2003

Kullback, S. & Leibler, R. A. 1951. On information and sufficiency. The annals of mathematical statistics, 22, (1), 79-86, www.jstor.org/stable/2236703



Housam Khalifa Bashier Babiker & Randy Goebel 2017. Using KL-divergence to focus Deep Visual Explanation. arXiv preprint arXiv:1711.06431.



Housam Khalifa Bashier Babiker & Randy Goebel 2017. An Introduction to Deep Visual Explanation. arXiv preprint arXiv:1711.09482.

$$p_{ij} = \frac{(1 + \|k_i - k_j\|^2)^{-k}}{\sum_{a \neq b} (1 + \|k_a - k_b\|^2)^{-k}} \quad (1)$$

Here p_{ij} denotes the joint probabilities, k is the raw class scores before softmax, i indexes a neuron value and $\sum_{a \neq b}$ combines all the values. For the ground truth we estimate the pairwise affinities with perplexity. We then compute the KL-divergence gradient i.e. $\frac{\partial E}{\partial z} \Rightarrow z$ derived here [6]. We also normalize the gradient to a zero mean and unit variance as follows:

$$\alpha_i = \frac{z_i - \mu}{\sigma} \quad (2)$$

The obtained weights α capture the relevant information in the feature maps acquired by the network. These weights are applied to every feature map $x_i \in X$ as to identify the discriminative pixels which influence the final prediction output as follows:

$$E_{KL-divergence} = \sum_i \sum_j x_i * |\alpha_j| \quad (3)$$

Housam Khalifa Bashier Babiker & Randy Goebel 2017. Using KL-divergence to focus Deep Visual Explanation. arXiv preprint arXiv:1711.06431.

Algorithm 1 Proposed approach

Input: image, ground truth y

Output: Discriminative localization map $\Rightarrow E_{KL-divergence}$

Apply a single forward-pass to estimate $\Rightarrow y$

Compute the joint probabilities for both y and \hat{y}

Compute the gradient and normalize using (2) $\Rightarrow \alpha$

initialize $E_{KL-divergence}$ to zero

for $i = 1$ **to** nFeatureMaps **do**

 Initialize temp to zero

for $j = 1$ **to** nClasses **do**

 temp \leftarrow temp + $(z_i * |\alpha_j|)$

end for

$E_{KL-divergence} \leftarrow E_{KL-divergence} + \text{temp}$

end for

Housam Khalifa Bashier Babiker & Randy Goebel 2017. Using KL-divergence to focus Deep Visual Explanation. arXiv preprint arXiv:1711.06431.

$$H[x] = - \sum_{\mathbf{x}} p(\mathbf{x}) \log_2 p(\mathbf{x})$$

Shannon, C. E. 1948. A Mathematical Theory of Communication. Bell System Technical Journal, 27, 379-423.

Important quantity in

- coding theory
- statistical physics
- machine learning

$$H[y|x] = - \iint p(y, x) \ln p(y|x) dy dx$$

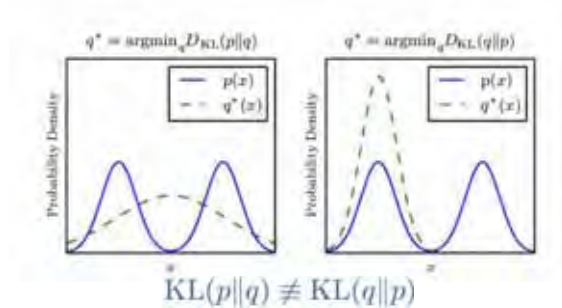
$$H[x, y] = H[y|x] + H[x]$$

$$\begin{aligned} \text{KL}(p||q) &= - \int p(\mathbf{x}) \ln q(\mathbf{x}) d\mathbf{x} - \left(- \int p(\mathbf{x}) \ln p(\mathbf{x}) d\mathbf{x} \right) \\ &= - \int p(\mathbf{x}) \ln \left\{ \frac{q(\mathbf{x})}{p(\mathbf{x})} \right\} d\mathbf{x} \end{aligned}$$

$$\text{KL}(p||q) \simeq \frac{1}{N} \sum_{n=1}^N \{-\ln q(\mathbf{x}_n|\theta) + \ln p(\mathbf{x}_n)\}$$

$$\text{KL}(p||q) \geq 0$$

KL-divergence is often used to measure the distance between two distributions



Goodfellow, I., Bengio, Y. & Courville, A. 2016. Deep Learning, Cambridge (MA), MIT Press.

- ... are **robust** against noise;
- ... can be applied to **complex time series** with good replication;
- ... is **finite** for stochastic, noisy, composite processes;
- ... the values correspond directly to irregularities – good for detecting **anomalies**

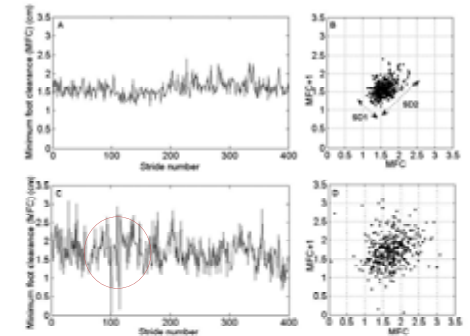
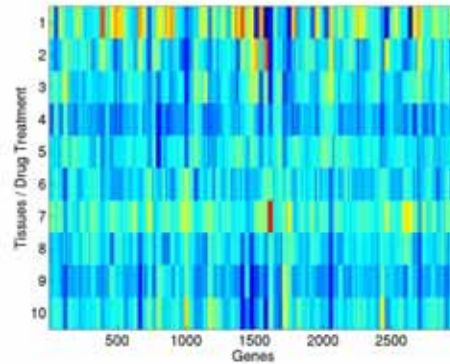


Thank you!

Questions

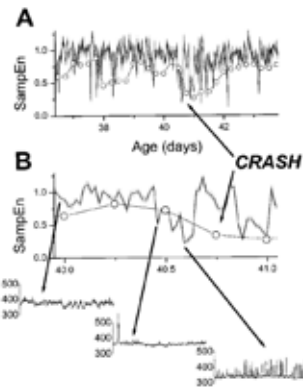


Thank you!



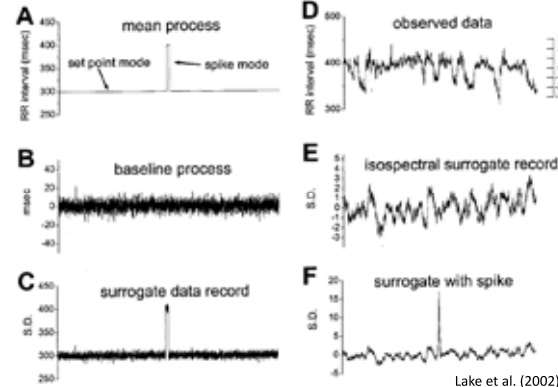
Khandoker, A., Palaniswami, M. & Begg, R. (2008) A comparative study on approximate entropy measure and poincare plot indexes of minimum foot clearance variability in the elderly during walking. *Journal of NeuroEngineering and Rehabilitation*, 5, 1, 4.

Back-up Slide: SampEn (1/2)



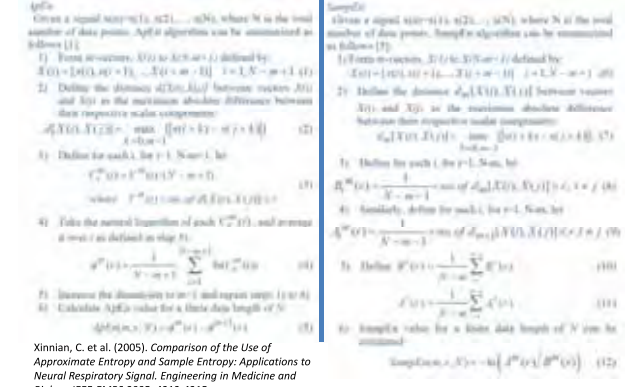
Lake, D. E., Richman, J. S., Griffin, M. P. & Moorman, J. R. (2002) Sample entropy analysis of neonatal heart rate variability. *American Journal of Physiology-Regulatory Integrative and Comparative Physiology*, 283, 3, R789-R797.

Backup: SampEn (2/2) Surrogate data heart rate variability



Lake et al. (2002)

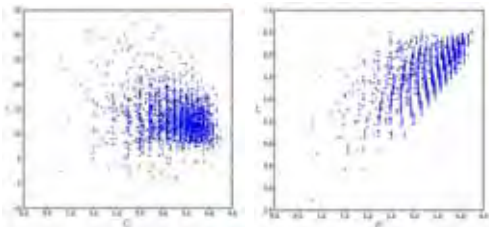
Backup Slide: Comparison ApEn - SampEn



Xinnian, C. et al. (2005). Comparison of the Use of Approximate Entropy and Sample Entropy: Applications to Neural Respiratory Signal. *Engineering in Medicine and Biology IEEE-EMBS 2005*, 4212-4215.

Backup Slide: Graph Entropy Measures

- The most important question: Which kind of structural information does the entropy measure detect?
- the topological complexity of a molecular graph is characterized by its number of vertices and edges, branching, cyclicity etc.



Dehmer, M. & Mowshowitz, A. (2011) A history of graph entropy measures. *Information Sciences*, 181, 1, 57-78.

Backup: English/German Subject Codes OEFOS 2012

106005	Bioinformatics	Bioinformatik
106007	Biostatistics	Biostatistik
304005	Medical Biotechnology	Medizinische Biotechnologie
305901	Computer-aided diagnosis and therapy	Computerunterstützte Diagnose und Therapie
304003	Genetic engineering, -technology	Gentechnik, -technologie
3906 (old)	Medical computer sciences	Medizinische Computerwissenschaften
305906	Medical cybernetics	Medizinische Kybernetik
305904	Medical documentation	Medizinische Dokumentation
305905	Medical informatics	Medizinische Informatik
305907	Medical statistics	Medizinische Statistik

<http://www.statistik.at>

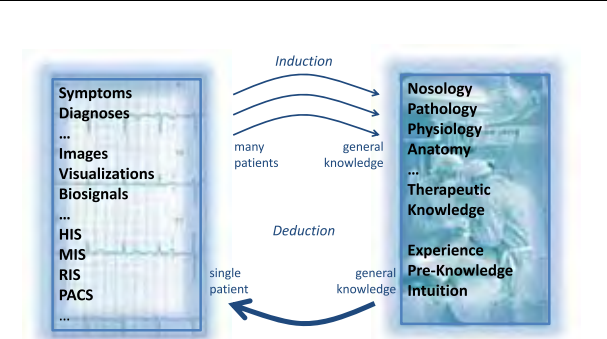
Backup: English/German Subject Codes OEFOS 2012

102001	Artificial Intelligence	Künstliche Intelligenz
102032	Computational Intelligence	Computational Intelligence
102033	Data Mining	Data Mining
102013	Human-Computer Interaction	Human-Computer Interaction
102014	Information design	Informationsdesign
102015	Information systems	Informationssysteme
102028	Knowledge engineering	Knowledge Engineering
102019	Machine Learning	Maschinelles Lernen
102020	Medical Informatics	Medizinische Informatik
102021	Pervasive Computing	Pervasive Computing
102022	Software development	Softwareentwicklung
102027	Web engineering	Web Engineering

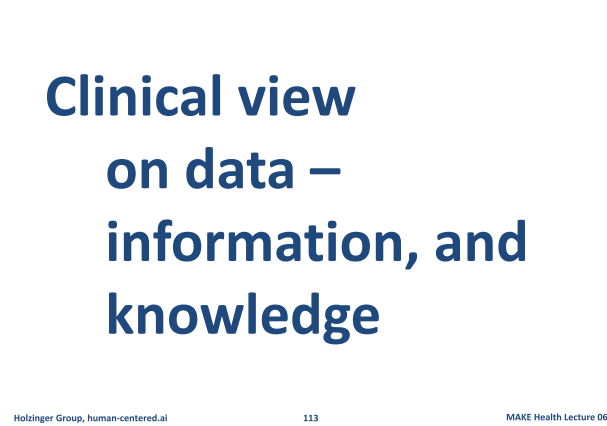
<http://www.statistik.at>

- Abduction** = cyclical process of generating possible explanations (i.e., identification of a set of hypotheses that are able to account for the clinical case on the basis of the available data) and testing those (i.e., evaluation of each generated hypothesis on the basis of its expected consequences) for the abnormal state of the patient at hand;
- Abstraction** = data are filtered according to their relevance for the problem solution and chunked in schemas representing an abstract description of the problem (e.g., abstracting that an adult male with haemoglobin concentration less than 14g/dL is an anaemic patient);
- Artefact/surrogate** = error or anomaly in the perception or representation of information through the involved method, equipment or process;
- Data** = physical entities at the lowest abstraction level which are, e.g. generated by a patient (patient data) or a (biological) process; data contain no meaning;
- Data quality** = Includes quality parameter such as : Accuracy, Completeness, Update status, Relevance, Consistency, Reliability, Accessibility;
- Data structure** = way of storing and organizing data to use it efficiently;
- Deduction** = deriving a particular valid conclusion from a set of general premises;
- DIK-Model** = Data-Information-Knowledge three level model
- DIKW-Model** = Data-Information-Knowledge-Wisdom four level model
- Disparity** = containing different types of information in different dimensions
- Heart rate variability (HRV)** = measured by the variation in the beat-to-beat interval;
- HRV artifact** = noise through errors in the location of the instantaneous heart beat, resulting in errors in the calculation of the HRV, which is highly sensitive to artifact and errors in as low as 2% of the data will result in unwanted biases in HRV calculations;

- ApEn = Approximate Entropy;
- \mathbb{C}_{data} = Data in computational space;
- DIK = Data-Information-Knowledge-3-Level Model;
- DIKW = Data-Information-Knowledge-Wisdom-4-Level Model;
- GraphEn = Graph Entropy;
- H = Entropy (General);
- HRV = Heart Rate Variability;
- MaxEn = Maximum Entropy;
- MinEn = Minimum Entropy;
- NE = Normalized entropy (measures the relative informational content of both the signal and noise);
- \mathbb{P}_{data} = Data in perceptual space;
- PDB = Protein Data Base;
- SampEn = Sample Entropy;

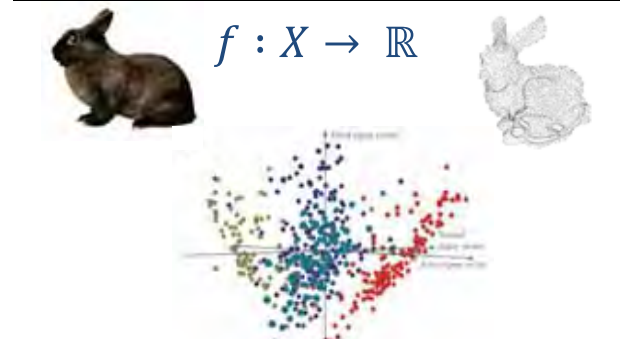
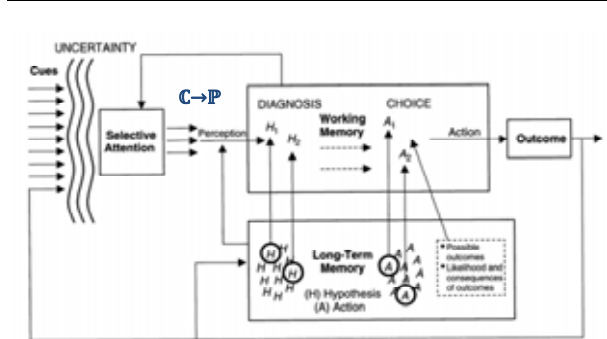
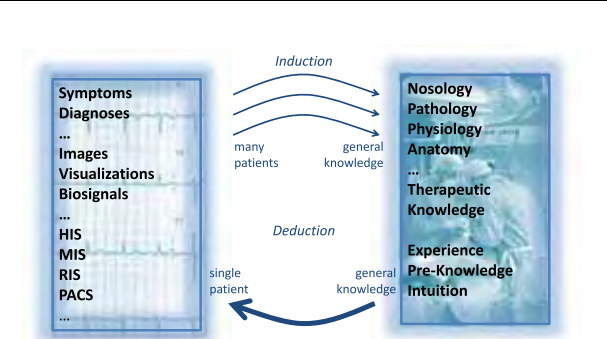
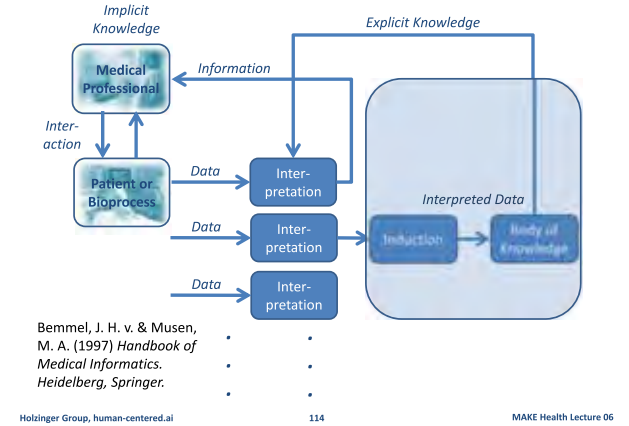


- Induction** = deriving a likely general conclusion from a set of particular statements;
- Information** = derived from the data by interpretation (with feedback to the clinician);
- Information Entropy** = a measure for uncertainty: highly structured data contain low entropy, if everything is in order there is no uncertainty, no surprise, ideally $H = 0$
- Knowledge** = obtained by inductive reasoning with previously interpreted data, collected from many similar patients or processes, which is added to the "body of knowledge" (explicit knowledge). This knowledge is used for the interpretation of other data and to gain implicit knowledge which guides the clinician in taking further action;
- Large Data** = consist of at least hundreds of thousands of data points
- Multi-Dimensionality** = containing more than three dimensions and data are multi-variate
- Multi-Modality** = a combination of data from different sources
- Multivariate** = encompassing the simultaneous observation and analysis of more than one statistical variable;
- Reasoning** = process by which clinicians reach a conclusion after thinking on all facts;
- Spatiality** = contains at least one (non-scalar) spatial component and non-spatial data
- Structural Complexity** = ranging from low-structured (simple data structure, but many instances, e.g., flow data, volume data) to high-structured data (complex data structure, but only a few instances, e.g., business data)
- Time-Dependency** = data is given at several points in time (time series data)
- Voxel** = volumetric pixel = volumetric picture element

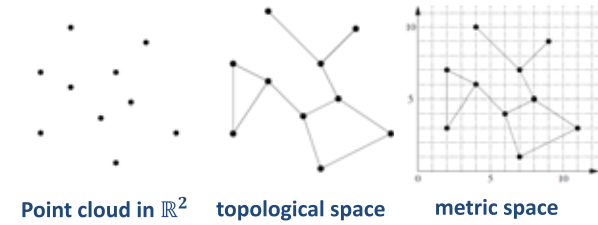


"In mathematics you don't understand things. You just get used to them" – John von Neumann

<p>Data</p> <p>n Number of samples</p> <p>d Number of input variables</p> <p>$\mathbf{X} = [\mathbf{x}_1, \dots, \mathbf{x}_n]$ Matrix of input samples</p> <p>$\mathbf{y} = [y_1, \dots, y_n]$ Vector of output samples</p> <p>$\mathbf{Z} = [\mathbf{x}, \mathbf{y}]$ Combined input-output training data or Representation of data points in a feature space</p>	
<p>Distribution</p> <p>P Probability</p> <p>$F(\mathbf{x})$ Cumulative probability distribution function (cdf)</p> <p>$p(\mathbf{x})$ Probability density function (pdf)</p> <p>$p(\mathbf{x}, \mathbf{y})$ Joint probability density function</p> <p>$p(\mathbf{x} \omega)$ Probability density function, which is parameterized</p> <p>$p(\mathbf{y} \mathbf{x})$ Conditional density</p> <p>$\eta(\mathbf{x})$ Target function</p>	



Let us collect n -dimensional i observations: $\mathbf{x}_i = [x_{i1}, \dots, x_{in}]$



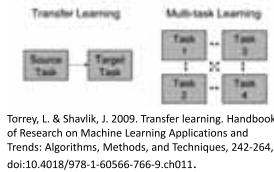
Zomorodian, A. J. 2005. *Topology for computing*, Cambridge (MA), Cambridge University Press.

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118

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- Big data with many training sets (this is good for ML!)
- Small number of data sets, rare events
- Very-high-dimensional problems
- Complex data – NP-hard problems
- Missing, dirty, wrong, noisy, ..., data
- GENERALISATION
- TRANSFER



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121

MAKE Health Lecture 06



"My greatest concern was what to call it. I thought of calling it "information", but the word was overly used, so I decided to call it "uncertainty". When I discussed it with John von Neumann, he had a better idea. Von Neumann told me, "You should call it entropy, for two reasons. In the first place your uncertainty function has been used in statistical mechanics under that name, so it already has a name. In the second place, and more important, nobody knows what entropy really is, so in a debate you will always have the advantage."

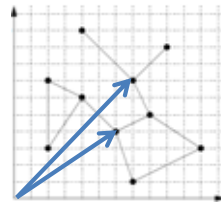
Tribus, M. & McIrvine, E. C. (1971) Energy and Information. *Scientific American*, 225, 3, 179-184.

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124

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A set S with a metric function d is a metric space



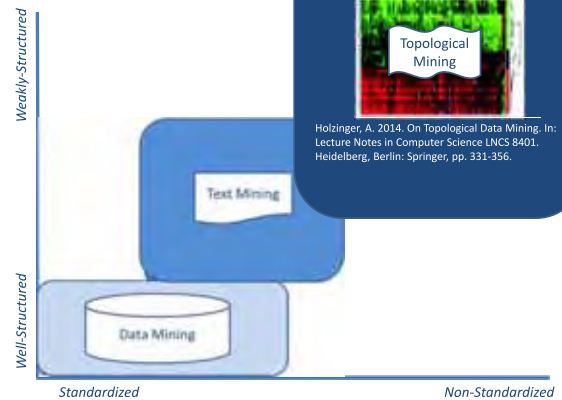
$$d_{ij} = \sqrt{\sum_{k=1}^p (x_{ik} - x_{jk})^2}$$

Doob, J. L. 1994. *Measure theory*, Springer New York.

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119

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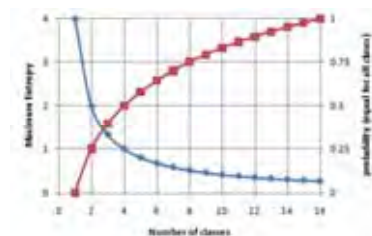


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122

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$$\log_2 \frac{1}{p} = -\log_2 p \quad H = -\sum_{i=1}^N p_i \log_2(p_i)$$

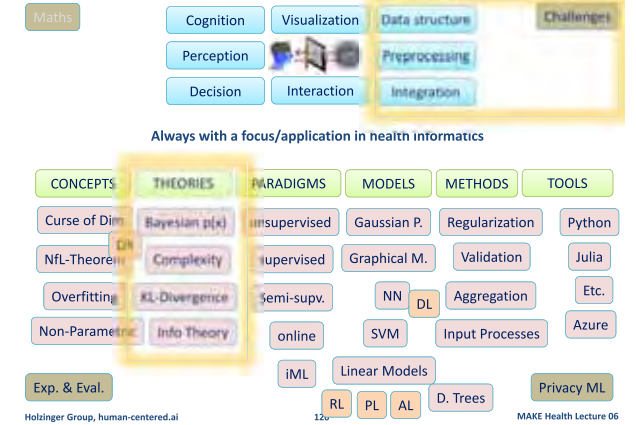


Shannon, C. E. (1948) A Mathematical Theory of Communication. *Bell System Technical Journal*, 27, 379-423.

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125

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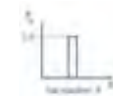


- $X: S \rightarrow \mathbb{R}$ ("measure" of outcome)
- Events can be defined according to X
 - $E(X=a) = \{s_i | X(s_i)=a\}$
 - $E(X \geq a) = \{s_i | X(s_i) \geq a\}$
- Consequently, probabilities can be defined on X
 - $P(X=a) = P(E(X=a))$
 - $P(a \geq X) = P(E(a \geq X))$
- partitioning the sample space

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$$H_B = -\sum_{k=1} p_k \log_2 p_k = -1 * \log_2(1) = 0$$



$$H_B = -\sum_{k=1}^B \frac{1}{B} \log_2 \frac{1}{B} = \log_2(B)$$



$$H = H_{max} = \log_2 N$$

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126

MAKE Health Lecture 06

- Developed by Claude Shannon in the 1940s
- Maximizing the amount of information that can be transmitted over an imperfect communication channel
- Data compression (entropy)
- Transmission rate (channel capacity)

Claude E. Shannon: *A Mathematical Theory of Communication*, *Bell System Technical Journal*, Vol. 27, pp. 379–423, 623–656, 1948

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Entropic methods – what for?

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- 1) Set of noisy, complex data
- 2) Extract information out of the data
- 3) to support a previous set hypothesis
- Information + Statistics + Inference
- = powerful methods for many sciences
- Application e.g. in biomedical informatics for analysis of ECG, MRI, CT, PET, sequences and proteins, DNA, topography, for modeling etc. etc.

Mayer, C., Bachler, M., Hortenhuber, M., Stocker, C., Holzinger, A. & Wassertheurer, S. 2014. Selection of entropy-measure parameters for knowledge discovery in heart rate variability data. *BMC Bioinformatics*, 15, (Suppl 6), S2.

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Categorization of Data (Classic “scales”)

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Scale	Empirical Operation	Mathem. Group Structure	Transf. in \mathbb{R}	Basic Statistics	Mathematical Operations
NOMINAL	Determination of equality	Permutation $x' = f(x)$ $x \dots 1\text{-to-1}$	$x \mapsto f(x)$	Mode, contingency correlation	$=, \neq$
ORDINAL	Determination of more/less	Isotonic $x' = f(x)$ $x \dots \text{mono-tonic incr.}$	$x \mapsto f(x)$	Median, Percentiles	$=, \neq, >, <$
INTERVAL	Determination of equality of intervals or differences	General linear $x' = ax + b$	$x \mapsto rx+s$	Mean, Std.Dev. Rank-Order Corr., Prod.-Moment Corr.	$=, \neq, >, <, -, +$
RATIO	Determination of equality or ratios	Similarity $x' = ax$	$x \mapsto rx$	Coefficient of variation	$=, \neq, >, <, -, +, \cdot, /$

Stevens, S. S. (1946) On the theory of scales of measurement. *Science*, 103, 677-680.



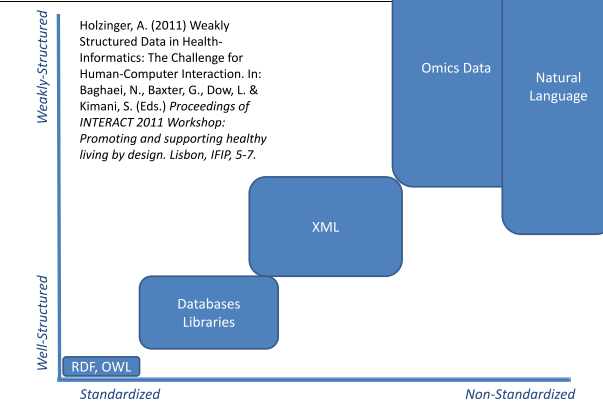
- The VC dimension is a measure of the capacity of a space of functions that can be learned by a statistical classification algorithm. It is defined as the cardinality of the largest set of points that the algorithm can shatter. It is a core concept in Vapnik–Chervonenkis theory

Vapnik, V. N. & Chervonenkis, A. Y. 1971. On the Uniform Convergence of Relative Frequencies of Events to Their Probabilities. *Theory of Probability & Its Applications*, 16, (2), 264-280, doi:10.1137/1116025.

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Standardization versus Structurization

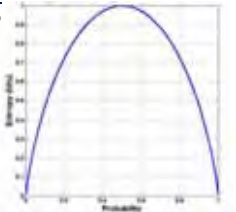
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$$Q \dots P = \{p_1, \dots, p_n\} \quad H(Q) = - \sum_{i=1}^n (p_i * \log p_i)$$

$$Qb = \{a_1, a_2\} \text{ with } P = \{p, 1 - p\}$$

$$H(Qb) = p * \log \frac{1}{p} + p * \log \frac{1}{1 - p}$$



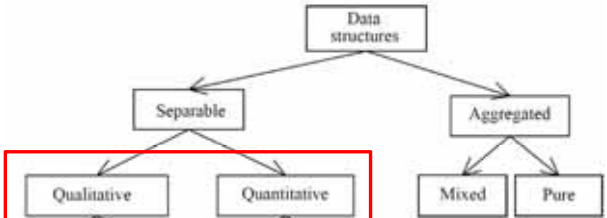
Shannon, C. E. (1948) A Mathematical Theory of Communication. *Bell System Technical Journal*, 27, 379-423.

Shannon, C. E. & Weaver, W. (1949) *The Mathematical Theory of Communication*. Urbana (IL), University of Illinois Press.

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Example: Data structures - Classification

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Aggregated attribute = a homomorphic map \mathcal{H} from a relational system $\langle A; \approx \rangle$ into a relational system $\langle B; = \rangle$; where A and B are two distinct sets of data elements.

Dastani, M. (2002) The Role of Visual Perception in Data Visualization. *Journal of Visual Languages and Computing*, 13, 601-622.

This is in contrast with other attributes since the set B is the set of data elements instead of atomic values.